PROCEEDINGS

OF

THE PHYSICAL SOCIETY

OF LONDON.

FEBRUARY 1886.

XXXIV. *Some Thermodynamical Relations.*—PART II.

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IN a note at the conclusion of the first of this series of papers, it was stated that the absolute temperatures of certain nearly related bodies corresponding to equal vapour-pressures are constant at all pressures. Further investigation, however, has shown that although it is only in the case of nearly allied substances, such as chlorobenzene and bromobenzene, that the ratio of the absolute temperatures corresponding to the same vapour-pressure is a constant, whatever (within the limits afforded by experimental data) that pressure may be, yet a relation does exist between the ratios of the absolute temperatures of all bodies, whether solid or liquid and whether stable or dissociable, which may be expressed in the case of any two bodies by the equation

$$R' = R + c(t' - t),$$

where R is the ratio of the absolute temperatures of the two bodies corresponding to any vapour-pressure, the same for both; R′ is the ratio at any other pressure, again the same for both; c is a constant which may be 0 or a small + or − number; and t' and t are the temperatures of one of the bodies corresponding to the two vapour-pressures.

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[*] Read December 12, 1885.

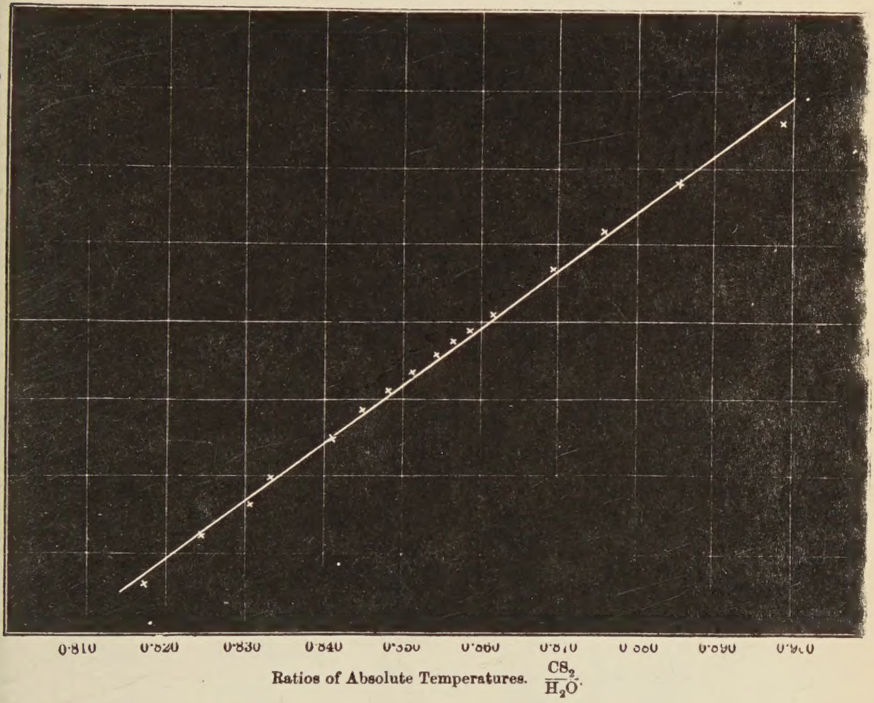
When $c=0$, $R'=R$, or the ratio of the absolute temperatures is a constant at all pressures; and where c is greater or less than 0, its value may readily be determined either by calculation, or graphically by representing the (absolute) temperatures of one of the two bodies as ordinates, and the ratio of the absolute temperatures at pressures corresponding to the absolute temperatures of that body as abscissæ. It is found that in all cases the points representing the relation of the ratio of the absolute temperatures of the two bodies to the absolute temperatures of one of them fall in a straight line. This is illustrated in some of the examples which are brought forward to prove the truth of the law. It follows from this that, if we know accurately the vapour-pressures of one substance, we only require two, or, better, three, accurate determinations of the vapour-pressure of any other substance, at temperatures moderately far apart, in order to be able to calculate the vapour-pressure of that substance at any required temperature, or, rather, to calculate the temperature corresponding to any vapour-pressure within the limits of pressure comprised in the determinations of the standard substance.

In the examples which follow, the same substance has not always been taken as the standard; but generally, in the comparison of any two bodies, that one is taken as correct which is likely to have been the more accurately investigated. Twenty-two different bodies have been examined, and they are arranged in twenty-three couples. The absolute temperatures corresponding to definite vapour-pressures of a number of substances are given in the tables in a previous portion of this paper. For other substances, curves were drawn to represent the relation of temperature to pressure, and the temperatures corresponding to definite pressures read off.

1. *Ratios of the Absolute Temperatures of Water and Carbon Bisulphide at Definite Vapour-pressures.*—As an example of the method employed, the whole of the data are given in this case, and also a diagram showing how the value of c may be obtained graphically. The absolute temperatures of water corresponding to the definite vapour-pressures are taken

to be correct, and are made the ordinates in fig. 1. The abscissæ are the ratios of the absolute temperatures of carbon bisulphide to those of water at those vapour-pressures. As

Fig. 1.



might be expected, some of the points representing the relation of the ratios of the absolute temperatures to the temperatures of water fall slightly to one or other side of the straight line drawn to pass through them. In order to find the error involved in these slight deviations from the straight line, points on the line itself corresponding to the same temperatures of water were read off, and these are given in the table as smoothed ratios. Lastly, the absolute temperatures of carbon bisulphide were calculated by multiplying those of water by the smoothed ratios; and the recalculated temperatures of carbon bisulphide are given together with the observed temperatures. It will be seen that the difference between the observed and recalculated temperatures is very small.

Pressures.	Absolute temperatures of water.	Ratios of absolute temperatures of CS_2 to those of H_2O.	Smoothed ratios.	Recalculated absolute temperatures of CS_2.	Observed absolute temperatures of CS_2.
millim.	°			°	°
50	311·3	·8161	·8160	254·0	254·05
100	324·7	·8245	·8242	267·6	267·7
150	333·1	·8301	·8296	276·3	276·5
200	339·6	·8339	·8338	283·2	283·2
300	348·9	·8403	·8400	293·1	293·2
400	356·0	·8448	·8449	300·8	300·75
500	361·7	·8485	·8483	306·8	306·9
600	366·5	·8517	·8519	312·2	312·15
700	370·7	·8545	·8545	316·8	316·75
800	374·45	·8567	·8571	320·9	320·8
900	377·8	·8589	·8590	324·5	324·5
1000	380·85	·8612	·8611	327·95	328·0
1500	393·2	·8695	·8692	341·8	341·9
2000	402·5	·8753	·8757	352·5	352·3
3000	416·5	·8852	·8850	368·6	368·7
5000	435·85	·8987	·8978	391·3	391·7

In the equation $R' = R + c(t' - t)$ the value of $c = ·0006568$. The greatest difference between the observed and recalculated temperatures is at the highest pressure (5000 millim.), and is only 0°·4.

2. *Water and Ethyl Alcohol.* $\frac{H_2O}{C_2H_6O}$. Temperatures of water taken as correct, and made the ordinates. $c = ·0000962$.

Pressures.	Ratios.	Smoothed ratios.	Absolute temp. of Alcohol.	
			Recalculated.	Observed.
10	1·058	1·0527	270°·1	269°·25
50	1·0553	1·0553	295·0	295·0
100	1·0559	1 0567	307·3	307·5
150	1·0565	1·0574	315·0	315·3
200	1·0580	1·0580	321·0	321·0
300	1·0579	1·0589	329·5	329·8
400	1·0595	1·0596	336·0	336·0
500	1·0604	1·0602	341·2	341·1
600	1·0609	1·0606	345·55	345·45
700	1·0619	1·0610	349·4	349·25
800	1·0618	1·0614	352·8	352·65
900	1·0624	1·0617	355·8	355·6
1000	1·0626	1·0620	358·6	358·4
1500	1·0633	1·0632	369·8	369·8
2000	1·0648	1·0641	378·25	378·0
3000	1·0663	1·0655	390·9	390·6
5000	1·0675	1·0673	408·4	408·3
10000	1·0687	1·0701	435·15	435·7
15000	1·0701	1·0720	452·7	453·5
20000	1·0715	1·0735	466·2	467·1

The only noticeable differences between the observed and recalculated temperatures are at the three highest pressures and at 10 millim. With regard to the error at the low pressure, it may be mentioned that a number of careful determinations of the vapour-pressure of alcohol at low temperatures were made by Ramsay and Young by the still method (see note at end of paper), and that their results differed slightly from those of Regnault. At a pressure of 50 millim. the difference was only 0°·05, but at 10 millim. the absolute temperature found by them was 270°·25, which is practically the same as that recalculated by means of the equation

$$R' = R + c(t' - t).$$

3. *Ethyl Alcohol and Methyl Alcohol.* $\frac{C_2H_6O}{CH_4O}$. Temperatures of ethyl alcohol taken as correct. $c = ·0001603$.

Pressures.	Ratios.		Absolute temperatures of Methyl Alcohol.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.			°	°
10	*a* 1·0372	1·0475	256·5	259·0
10	*b* 1·0428	1·0473	257·9	
50	1·0424	1·0433	282·8	283·0
100	1·0424	1·0414	295·3	295·1
150	1·0406	1·0400	303·2	303·0
200	1·0392	1·0392	308·9	308 9
300	1·0384	1·0379	317·75	317·6
400	1·0371	1·0368	324·1	324·0
500	1·0358	1·0359	329·3	329·3
600	1·0346	1·0351	333·75	333·9
700	1·0335	1·0346	337·4	337·8
800	1·0339	1·0340	341·05	341·1
900	1·0334	1·0336	344·05	344·1
1000	......	1·0331	346·9	......
1500	1·0304	1·0314	358·5	358·9
2000	1·0297	1·0300	367·0	367·1
3000	1·0276	1·0280	380·0	380·1
5000	1·0256	1·0251	398·3	398 1

Two values are given for 10 millim.:—(*a*) the ratio calculated from Regnault's values, and (*b*) the ratio obtained by employing the recalculated temperature of ethyl alcohol. The only noticeable difference between the observed and recalculated temperatures is at a pressure of 10 millim.; and by taking the recalculated temperature of ethyl alcohol it is greatly reduced.

4. *Carbon Bisulphide and Ethyl Oxide.* $\frac{CS_2}{C_4H_{10}O}$. Temperatures of carbon bisulphide taken as correct. $c = \cdot 0001621$.

Pressures.	Ratios.		Absolute temp. of Ethyl Oxide.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
100	1·0284	1·0283	260°·3	260°·3
150	1·0298	1·0298	268·5	268·5
200	1·0306	1·0307	274·8	274·8
300	1·0324	1·0325	284·0	284·0
400	1·0335	1·0337	290·95	291·0
500	1·0344	1·0346	296·65	296·7
600	1·0353	1·0355	301·45	301·5
700	1·0361	1·0361	305·7	305·7
800	1·0372	1·0369	309·4	309·3
900	1·0371	1·0375	312·8	312·9
1000	1·0380	1·0380	316·0	316·0
1500	1·0408	1·0402	328·7	328·5
2000	1·0423	1·0420	338·1	338·0
3000	1·0451	1·0446	352·95	352·8
5000	1·0490	1·0484	373·6	373·4

In this series the greatest difference between the observed and recalculated temperatures is only 0°·2.

5. *Carbon Bisulphide and Ethyl Bromide.* $\frac{CS_2}{C_2H_5Br}$. Temperatures of carbon bisulphide taken as correct. $c = \cdot 0001185$.

Pressures.	Ratios.		Absolute temperatures of Ethyl Bromide.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·0162	1·0172	249°·75	250°·0
100	1·0186	1·0188	262·8	262·8
150	1·0203	1·0199	271·1	271·0
200	1·0224	1·0206	277·5	277·0
300	1·0223	1·0218	286·9	286·8
400	1·0233	1·0228	294·05	293·9
500	1·0237	1·0234	299·9	299·8
600	1·0244	1·0241	304·8	304·7
700	1·0251	1·0247	309·1	309·0
800	1·0249	1·0251	312·9	313·0
900	1·0259	1·0256	316·4	316·3
1000	1·0263	1·0260	319·7	319·6
1500	1·0274	1·0276	332·7	332·8
2000	1·0280	1·0289	342·4	342·7
3000	1·0299	1·0308	357·7	358·0
5000	1·0327	1·0335	379·0	379·3

6. *Carbon Bisulphide and Ethyl Chloride.* $\frac{CS_2}{C_2H_5Cl}$. Temperatures of carbon bisulphide taken as correct. $c = {\cdot}0001050$.

Pressures.	Ratios.		Absolute temperatures of Ethyl Chloride.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
150	1·1118	1·1135	248°·3	248°·7
200	1·1136	1·1142	254·2	254·3
300	1·1152	1·1152	262·9	262·9
400	1·1164	1·1160	269·5	269·4
500	1·1164	1·1166	274·9	274·9
600	1·1174	1·1172	279·4	279·3
700	1·1181	1·1177	283·4	283·3
800	1·1178	1·1181	286·9	287·0
900	1·1186	1·1185	290·1	290·1
1000	1·1194	1·1189	293·15	293·0
1500	1·1210	1·1203	305·2	305·0
2000	1·1209	1·1215	314·1	314·3
3000	1·1224	1·1232	328·25	328·5
5000	1·1253	1·1256	348·0	348·1

7. *Chlorobenzene and Water.* $\frac{H_2O}{C_6H_5Cl}$. Temp. of water taken as correct. $c = {\cdot}000598$.

Pressures.	Ratios.		Absolute temperatures of Chlorobenzene.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
100	1·0579	1·0573	343°·3	343°·5
150	1·0625	1·0623	353·85	353·9
200	1·0654	1·0660	362·0	361·8
300	1·0711	1·0716	373·9	373·7
400	1·0753	1·0760	383·05	382·8
500	1·0788	1·0791	390·3	390·2
600	1·0819	1·0820	396·55	396·5
700	1·0845	1·0845	402·0	402·0

8. *Bromobenzene and Water.* $\frac{C_6H_5Br}{H_2O}$. Temperatures of water taken as correct. $c = \cdot000609$.

Pressures.	Ratios.		Absolute temperatures of Bromobenzene.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·1156	1·1124	346°·3	347°·3
100	1·1220	1·1206	363·9	364·3
150	1·1255	1·1258	375·0	374·9
200	1·1284	1·1297	383·65	383·2
300	1·1344	1·1353	396·1	395·8
400	1·1388	1·1396	405·7	405·4
500	1·1427	1·1431	413·45	413·3
600	1·1460	1·1460	420·0	420·0
700	1·1489	1·1486	425·8	425·9
800	1·1516	1·1509	430·95	431·2

9. *Aniline and Water.* $\frac{C_6H_5NH_2}{H_2O}$. Temperatures of water taken as correct. $c = \cdot000345$.

Pressures.	Ratios.		Absolute temperatures of Aniline.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·2046	1·2048	375°·05	375°·0
100	1·2100	1·2094	392·7	392·9
150	1·2129	1·2124	403·85	404·0
200	1·2144	1·2145	412·45	412·4
300	1·2174	1·2176	424·8	424·75
400	1·2197	1·2201	434·4	434·2
500	1·2217	1·2221	442·1	441·9
600	1·2237	1·2239	448·55	448·5
700	1·2254	1·2253	454·2	454·25
800	1·2269	1·2266	459·3	459·4

10. *Methyl Salicylate and Water.* $\frac{C_6H_4(OH)CO\,O(CH_3)}{H_2O}$. Temperatures of water taken as correct. $c = \cdot 000459$.

Pressures.	Ratios.		Absolute temperatures of Methyl Salicylate.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·3026	1·3008	404°·9	405°·5
100	1·3067	1·3069	424·35	424·3
150	1·3101	1·3107	436·6	436·4
200	1·3118	1·3136	446·1	445·5
300	1·3170	1·3179	459·8	459·5
400	1·3204	1·3212	470·35	470·05
500	1·3235	1·3239	478·9	478·7
600	1·3261	1·3260	486·0	486·0
700	1·3283	1·3280	492·3	492·4
800	1·3302	1·3298	497·95	498·1

11. *Bromonaphthalene and Water.* $\frac{C_{10}H_7Br}{H_2O}$. Temperatures of water taken as correct. $c = \cdot 000612$.

Pressures.	Ratios.		Absolute temperatures of Bromonaphthalene.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
150	1·4587	1·4592	486°·05	485°·9
200	1·4623	1·4631	496·9	496·6
300	1·4689	1·4690	512·5	512·5
400	1·4733	1·4732	524·45	524·5
500	1·4766	1·4769	534·2	534·1
600	1·4797	1·4798	542·3	542·3
700	1·4823	1·4821	549·4	549·5
800	1·4846	1·4847	555·95	555·9

12. *Sulphur and Carbon Bisulphide.* $\frac{S}{CS_2}$. Temperatures of carbon bisulphide taken as correct. $c = -·0006845$.

Pressures.	Ratios.		Absolute temperatures of Sulphur.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
300	2·2783	2·2779	667°·9	668°·0
400	2·2733	2·2727	683·5	683·7
500	2·2681	2·2685	696·2	696·1
600	2·2650	2·2649	707·0	707·0
700	2·2607	2·2617	716·4	716·1
800	2·2588	2·2588	724·6	724·6
900	2·2558	2·2563	732·2	732·0
1000	2·2530	2·2540	739·3	739·0
1500	....	2·2444	767·4	......
2000	2·2373	2·2372	788·15	788·2
3000	2·2262	2·2262	820·8	820·8

The close agreement between the observed and recalculated temperatures of such a high-boiling substance as sulphur is very striking.

13. *Carbon Bisulphide and Ethylene.* $\frac{CS_2}{C_2H_4}$. Temperatures of carbon bisulphide taken as correct. $c = ·000274$.

Pressures.	Ratios.		Absolute temperatures of Ethylene.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·8212	1·8208	139°·5	139°·5
100	1·8242	1·8244	146·8	146·75
150	1·8263	1·8268	151·35	151·4
200	1·8283	1·8287	154·9	154·9
300	1·8325	1·8315	160·1	160·0
400	1·8339	1·8336	164·0	164·0

Here again the agreement is very remarkable, considering the difficulty of measuring very low temperatures accurately.

14. *Water and Oxygen.* $\frac{H_2O}{O}$. Temperatures of water taken as correct. $c = -·0003932$.

Vapour-pressures of oxygen have been determined by Olszewski and by Wroblewski, but their results do not agree well together. Olszewski measured his temperatures by means of a hydrogen-thermometer; Wroblewski by a thermopile. It was found that the curve showing the relation of temperature to pressure was very fairly regular when constructed from Olszewski's numbers, and the temperatures corresponding to definite pressures were therefore read from this curve. The temperature corresponding to a pressure of 9 millim. is a single observation by Olszewski. The temperatures corresponding to 50, 100, and 150 millim. are taken from Wroblewski's results.

Pressures.	Ratios.		Absolute temperatures of Oxygen.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
9	4·5967	4·454	63°·5	61°·5
50	4·0534	4·349	71·6	76·8
100	3·9358	4·2845	75·8	82·5
150	4·0132	4·253	78·6	83·0
300	....	4·191	83·2	......
400	....	4·182	85·1	......
500	....	4·1405	87·4	......
600	....	4·120	89·0	......
700	....	4·1025	90·4	......
800	4·0701	4·090	91·6	92·0
1000	4·0732	4·064	93·7	93·5
1500	4·0536	4·0175	97·9	97·0
2000	4·0520	3·980	101·1	100·0
3000	3·9478	3·924	106·1	105·5
5000	3·8267	3·849	113·2	113·9
10000	3·7044	3·733	124·7	125·7
15000	3·6489	3·656	132·7	133·0
20000	3·6242	3·5975	139·1	138·1

It is hardly to be expected that the measurement of such extremely low temperatures could be very exact, and the difference of 2°·0 at an absolute temperature of about 60°, is not more than might be expected. It will be seen that the three temperatures given by Wroblewski differ considerably from those calculated from Olszewski's results.

The vapour-pressures of oxygen are given up to the critical point, at which the pressure is 38,600 millim., but those of water have only been determined up to 20,000 millim. The absolute temperatures of oxygen were therefore compared with those of alcohol, in order to find whether the value of c remains constant up to the critical point.

15. *Alcohol and Oxygen.* $\frac{C_2H_6O}{O}$. Temperatures of alcohol taken as correct. $c = -·00040075$.

Pressures.	Ratios.		Absolute temperatures of Oxygen.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
300	...	3·940	83°·7	......
400	...	3·913	85·9	......
500	...	3·894	87·6	......
600	...	3·879	89·1	......
700	...	3·861	90·4	
800	3·8331	3·848	91·6	92°·0
1000	3·8332	3·824	93·7	93·5
1500	3·8123	3·780	97·8	97·0
2000	3·7800	3·7485	100·8	100·0
3000	3·7024	3·694	105·7	105·5
5000	3·5848	3·621	112·8	113·9
10000	3·4662	3·514	124·0	125·7
15000	3·4098	3·441	131·8	133·0
20000	3·3824	3·388	137·9	138·1
30000	3·3189	3·300	147·9	147·1
40000	3·2698	3·240	155·6	*154·2

The absolute temperatures of oxygen recalculated from the smoothed ratios with alcohol agree well with those calculated from the ratios with water. It was thought worth while to compare these temperatures with those of sulphur, oxygen being the most and sulphur the least volatile of all the substances examined.

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* The last pressure, 40,000 millim., is a little above the critical point, but sufficiently near to allow of the continuation of the vapour-pressure curve without sensible error. This was done instead of calculating the absolute temperature of alcohol corresponding to 38600 millim., the critical pressure of oxygen. It is to be remarked that the last two or three points representing the experimentally determined vapour-pressures of oxygen at the highest temperatures lie a little above the curve which was drawn. If more weight had been given to these points, the agreement between the observed and recalculated numbers would have been much closer. Thus Olszewski gives 154°·2 as the absolute temperature at the critical pressure 38600 millim., whereas from the curve this would be the temperature at 40000 millim. If the curve were raised so as to pass through the point representing the observed critical pressure and temperature, the exterpolated temperature would become 155 ·3, or only 0°·3 lower than that recalculated from the smoothed ratio.

In any case, the agreement seems to be close enough to show that the equation $R' = R + c\,(t' - t)$ holds good up to the critical point.

16. *Sulphur and Oxygen.* $\frac{S}{O}$. Temperatures of Sulphur taken as correct. $c = -{\cdot}0001652$.

Pressures.	Ratios.			Absolute Temperatures of Oxygen.				
	From observed temperatures.	From mean of recalculated temperatures.	Smoothed.	*a.* From $\frac{H_2O}{O}$.	*b.* From $\frac{C_2H_6O}{O}$.	*c.* Mean of *a* & *b*.	From $\frac{S}{O}$.	Observed.
millim. 300	...	8·005	8·005	83°·2	83°·7	83°·45	83°·45	...
400	...	7·9965	7·980	85·1	85·9	85·5	85·7	...
500	...	7·955	7·957	87·4	87·6	87·5	87·5	...
600	...	7·9395	7·9395	89·0	89·1	89·05	89·05	...
700	...	7·921	7·9245	90·4	90·4	90·4	90·4	...
800	7·876	7·910	7·910	91·6	91·6	91·6	91·6	92°·0
1000	7·904	7·887	7·8875	93·7	93·7	93·7	93·7	93·5
1500	7·9155	7·847	7·840	97·9	97·8	97·85	97·9	97·0
2000	7·882	7·808	7·8055	101·1	100·8	100·95	101·0	100·0
3000	7·780	7·750	7·7525	106·1	105·7	105·9	105·9	105·5

It will be seen that the temperatures of oxygen recalculated from those of sulphur, when $c = -·0001652$, agree remarkably well with the mean of the temperatures recalculated from the ratios with water and alcohol.

17. *Acetic Acid and Water.* $\frac{C_2H_4O_2}{H_2O}$. Temperatures of water taken as correct. For liquid acetic acid $c = ·0003186$; for solid acetic acid $c = -·000852$.

In the 'Philosophical Transactions,' Part II., 1884, it was shown experimentally by Ramsay and Young that the vapour-pressures of solid acetic acid were lower than those of the liquid acid at the same temperature, and curves were constructed showing the relation of temperature to pressure for both the solid and the liquid acid. The absolute temperatures corresponding to definite pressures were read from these curves. For pressures below 5 millim. the vapour-pressures of water

a. Liquid Acetic Acid and Water.

Pressures.	Ratios.		Absolute temperatures of Acetic Acid.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
5	1·0175	1·0188	279°·35	279°·01
6	1·0193	1·0195	282·2	282·14
7	1·0202	1·0202	284·6	284·64
8	1·0212	1·0209	286·8	286·91
9	1·0219	1·0215	288·8	288·88
10	1·0222	1·0220	290·55	290·6
50	1·0311	1·0305	320·8	321·0
100	1·0345	1·0347	336·0	335·9
150	1·0369	1·0375	345·6	345·4
200	1·0392	1·0395	353·0	352·9
300	1·0426	1·0424	363·7	363·75
400	1·0447	1·0447	371·9	371·9
500	1·0455	1·0465	378·5	378·15
600	1·0478	1·0480	384·1	384·0
700	1·0506	1·0495	389·05	389·45

b. Solid Acetic Acid and Water.

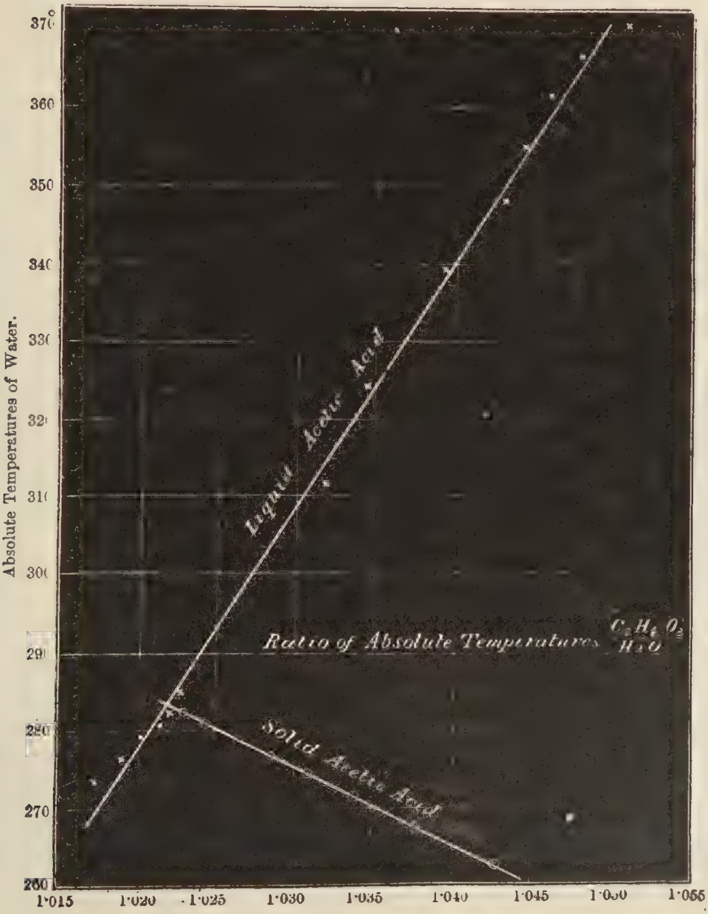
2	1·0423	1·0422	272·9	272·89
3	1·0372	1·0374	277·15	277·10
4	1·0335	1·0335	280·15	280·14
5	1·0306	1·0307	282·6	282·60
6	1·0284	1·0284	284·7	284·67
7	1·0264	1·0261	286·3	286·37
8	1·0244	1·0244	287·8	287·8
9	1·0226	1·0226	289·1	289·08

adopted by Regnault were not employed, the values calculated from his empirical formula representing the relation of temperature to pressure of water between 0° and 100° being probably more correct.

The differences between the recalculated and observed absolute temperatures of solid acetic acid are extremely minute. If the vapour-pressures of water given by Regnault were employed, the value of c would be slightly altered, and in that case the agreement would not be quite so close, the greatest difference amounting to 0°·15.

The results for both the solid and the liquid acid are shown in the diagram (fig. 2).

Fig. 2.



Dissociating Substances.

Five of these bodies have been examined, three liquid and two solid. In all of them the value of *c* is constant, while in the case of nitrogen peroxide, when compared with water it is$=0$, or at any rate it is a very small number.

18. *Nitrogen Peroxide and Water.* $\frac{N_2O_4}{H_2O}$. Water taken as correct. $c=0$.

Pressures.	Ratios.		Absolute temperatures of N_2O_4.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
100	0·7884	0·790	256°·5	256°·0
150	0·7899	„	263·15	263·1
200	0·7895	„	268·3	268·1
300	0·7908	„	275·6	275·9
400	0·7904	„	281·25	281·4
500	0·7904	„	285·75	285·9

Considering the nature of the substance, these differences may probably be considered within the limits of experimental error.

19. *Chloral Ethyl-alcoholate and Water.* $\frac{CCl_3CH\left\{\begin{matrix}OH\\OC_2H_5\end{matrix}\right.}{H_2O}$.

Temperatures of water taken as correct. $c=-\cdot0005119$.

Pressures.	Ratios.		Absolute temperatures of Chloral Ethyl-alcoholate.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·0729	1·0728	334°·0	334°·0
100	1·0659	1·0659	346·1	346·1
150	1·0619	1·0615	353·6	353·7
200	1·0545	1·0582	359·4	358·1
300	1·0530	1·0535	367·6	367·4
400	1·0500	1·0499	373·8	373·8
500	1·0470	1·0470	378·7	378·7

20. *Chloral Methyl-alcoholate and Water.* $\frac{CCl_3CH\left\{\begin{matrix}OH\\ OCH_3\end{matrix}\right.}{H_2O}$.

Temperatures of water taken as correct. $c = -{\cdot}0005392$.

Pressures.	Ratios.		Absolute temperatures of Chloral Methyl-alcoholate.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·0639	1·0641	331°·25	331°·2
100	1·0573	1·0570	343·2	343·3
150	1·0532	1·0546	351·3	350·8
200	1·0486	1·0489	356·2	356·1
300	1·0439	1·0439	364·2	364·2
400	1·0402	1·0400	370·2	370·3

The last two substances examined are solid throughout, and, at the higher temperatures at any rate, dissociate completely on being converted into vapour.

21. *Ammonium Chloride and Water.* $\frac{NH_4Cl}{H_2O}$. Temperatures of water taken as correct. $c = -{\cdot}0006667$.

Pressures.	Ratios.		Absolute temperatures of NH_4Cl.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.				
50	1·6855	1·6809	523°·3	524°·7
100	1·6717	1·6720	542·9	542·8
150	1·6650	1·6664	555·1	554·6
200	1·6587	1·6620	564·4	563·3
300	1·6546	1·6560	577·8	577·3
400	1·6514	1·6511	587·8	587·9
500	1·6478	1·6475	595·9	596·0
600	1·6453	1·6441	602·6	603·0

The determinations of the vapour-pressures and temperatures of volatilization of ammonium chloride were attended with great difficulties, and the agreement between the calculated and observed temperatures is as near as could be expected. The volatilizing-point at the atmospheric pressure, calculated from the equation $R' = R + c(t' - t)$, would be 338°·6 C., which agrees well enough with the temperature found by experiment.

22. *Ammonium Carbamate and Water.* $\frac{CO\left\{\begin{matrix}NH_2\\ONH_4\end{matrix}\right.}{H_2O}$. Temperatures of water taken as correct. $c = -{\cdot}000696$.

Pressures.	Ratios.		Absolute temperatures of Ammonium Carbamate.	
	From observed temperatures.	Smoothed.	Recalculated.	Observed.
millim.			°	°
10	0·9529	0·9510	270·4	270·9
50	0·9313	0·9321	290·2	289·9
100	0·9221	0·9230	299·7	299·4
150	0·9159	0·9170	305·45	305·1
200	0·9122	0·9126	309·9	309·8
300	0·9060	0·9061	316·15	316·1
400	0·9017	0·9011	320·8	321·0

It is thought that these twenty-two examples, including twenty-one different bodies—solid, liquid, stable, and dissociable—are sufficient to prove that the equation $R' = R + c(t' - t)$ is applicable to all classes of bodies, and that by the determination of the constant c, which involves only a small amount of experimental work, it becomes possible to calculate the vapour-pressures of any substance, assuming those of water or some other body to be accurately known, within the limits of pressure included in the experimental determinations of the standard substance.

It should be pointed out that those dissociable substances which have been investigated, are either, as in the case of ammonium carbamate and chloride and the compounds of chloral, wholly or almost wholly dissociated on their passage into the gaseous state at the temperatures of observation, or the amount of dissociation is very small, as with nitric peroxide. Until reliable data are obtained, it is perhaps premature to make any complete statement in reference to the behaviour of dissociable bodies.

On the Vapour-Pressures of Mercury.

It will have been observed in the tables in Part I. of this series of papers that mercury appears to differ from all the other substances examined, inasmuch as the values of $\frac{dp}{dt} \cdot t$, when compared with either water or carbon bisulphide, are not even approximately constant at different pressures, but rise steadily and somewhat rapidly. Again, the absolute temperatures of mercury cannot be calculated from those of water by means of the equation $R' = R + c(t' - t)$, for if Regnault's data are correct, c is not a constant. There appeared, however, to be good reason to believe that Regnault's values were not reliable, and he himself admits that great difficulties were experienced in the investigation of this substance.

It was, therefore, considered desirable to attempt a redetermination of the vapour-pressures of mercury with the greatest possible accuracy at a few temperatures. This has been accomplished, and the results are fully described in Trans. Chem. Soc. Jan. 1886. It will be sufficient here to mention that new determinations were made at the boiling-points of methyl salicylate and of sulphur under atmospheric pressure (220° and 450° approximately). Two other determinations at the boiling-point of bromonaphthalene under atmospheric and reduced pressure (280° and 270°), described in the Trans. Chem. Soc. xlvii. p. 640, and four determinations of the boiling-point of mercury under atmospheric pressure by Regnault (*Mémoires de l'Académie*, xxi. p. 230), were also made use of in the calculations.

On comparing the absolute temperatures of water with those of mercury at these five pressures, it was found that the value of c in the equation $R' = R + c(t' - t)$ was constant, and that mercury did not form an exception to the law, which has been proved to hold good for all the other substances examined.

By making the absolute temperatures of mercury ordinates, and the ratios of the absolute temperatures of mercury to those of water at the same pressures abscissæ, and drawing a straight line through the five experimentally-determined points, the value of c was ascertained, and vapour-pressures of mercury were calculated for each 5°, from 270° to 520° C. It should be mentioned that slightly different results were obtained when the absolute temperatures of water were made the ordinates; but within the limits of temperature just mentioned, the difference was unimportant.

With the absolute temperatures of mercury as ordinates, the value of c is ·0004788.

The values of $\frac{dp}{dt} \, . \, t$ for mercury were then determined; for the most part by the method of tangents, but between 150 and 700 mm. from the vapour-pressures calculated for each degree of temperature, and were compared with those of water and of carbon bisulphide. The results are given in the following table :—

Pressure.	$\frac{dp}{dt}$.	t.	$\frac{dp}{dt} \, . \, t$.	Water made =1·000	Carbon bi-sulphide made = 1·000
millim.					
10	0·350	457°·3	160	0·866	
50	1·405	508·5	714	0·850	1·057
100	2·506	534·2	1339	0·833	1·046
150	3·54	550·97	1950	0·843	1·061
200	4·47	563·44	2519	0·835	1·061
300	6·25	582·21	3639	0·834	1·061
400	7·85	596·44	4682	0·827	1·055
500	9·40	608·03	5715	0·827	1·055
600	10·91	617·87	6741	0·829	1·060
700	12·34	626·48	7731	0·824	1·057
800	13·75	634·0	8717	0·829	1·059
900	15·01	641·0	9621	0·814	1·037
1000	16·37	647·2	10595	0·818	1·049
1500	22·38	673·1	15064	0·811	1·038
2000	28·33	693·0	19633	0·816	1·064
3000	38·61	723·1	27919	0·804	1·071
5000	56·90	765·6	43562	0·800	1·059

It will be seen that the agreement with carbon bisulphide is extremely close throughout, and that with water the variation is not greater than with some of the other substances examined. The variations from constancy in the reduced values of $\frac{dp}{dt} \, . \, t$ will be considered in Part III.

Note.—Reference has been made to the vapour-pressures of alcohol at high and low temperatures determined by ourselves. The data are contained in a paper presented to the Royal Society in May 1885. The vapour-pressures of dissociable substances are given in a paper read before the Royal Society in November 1885. Neither of these papers is as yet published.

XXXV. *Some Thermodynamical Relations.*—PART III. *By* WILLIAM RAMSAY, *Ph.D., and* SYDNEY YOUNG, *D.Sc.*[*]

IN the first of this series of papers (Phil. Mag. December 1885) two relations between the vapour-pressures of different substances were described:—1. The product of the rate of increase of pressure per unit rise of temperature $\left(\frac{dp}{dt}\right)$ into the absolute temperature (t) is approximately the same for all bodies at the same pressure, but the differences are real, and are not due to errors of experiment or of calculation. 2. The rate of increase of this value $\frac{dp}{dt}.t$ with rise of pressure is very nearly the same for all bodies. This was shown in the tables by making the product $\frac{dp}{dt}.t$ for one substance equal to unity at each pressure, and reducing the values for other substances at the same pressure in the same ratio. It was pointed out that, at pressures between 150 or 200 millim. and 1500 or 2000 millim., the variations from constancy are very small, but that they are frequently greater at lower and at higher pressures; and it was suggested that these variations might possibly be due either to errors of experiment or of calculation.

In the second paper a relation was proved to exist between the absolute temperature of any two bodies corresponding to equal vapour-pressures, which may be expressed by the equation

$$R' = R + c\,(t' - t),$$

where R' is the ratio of the absolute temperatures of any two bodies at a given pressure, R the ratio at another given pressure, t' and t are the temperatures of one of the two bodies corresponding to those vapour-pressures, and c is a constant. The value of c may be determined graphically by making the temperatures (absolute or Centigrade) of one of the bodies

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[*] Read December 12, 1885.

(A) ordinates and the ratios of the absolute temperatures of the two bodies (A and B) at pressures corresponding to the temperatures of (A) abscissæ, when the points representing this relation fall in a straight line.

In the case of nearly related bodies such as chlorobenzene and bromobenzene, $c=0$ and $R'=R$ at all pressures, and in those cases the ratios of the products $\frac{dp}{dt} \cdot t$ are also equal at all pressures; and it appeared likely that if the variations from constancy of the reduced values of $\frac{dp}{dt} \cdot t$ for other substances were real, they would be related in some way to the variations from constancy of the ratios of the absolute temperature. This is indeed the case; for if we compare the values of $\frac{dp}{dt} \cdot t$ for any two substances, and make the temperatures of one of them ordinates, and the ratios of the products $\frac{dp}{dt} \cdot t$ abscissæ, we again find that the points representing the relation of the ratios of the product $\frac{dp}{dt} \cdot t$ at definite pressures to the temperatures of one of the bodies corresponding to those pressures, fall in a straight line. We therefore obtain a similar equation,

$$R'=R+c\,(t'-t),$$

R' and R in this case representing the ratios of the products $\frac{dp}{dt} \cdot t$. The cause of the apparently greater deviation from constancy at low and at high pressures is simply this—that at low pressures a small change of pressure corresponds to a large change of temperature, while at high pressures the intervals of pressure taken are very great, and therefore the intervals of temperature are also great. This is clearly seen by referring to the table of absolute temperatures corresponding to the pressures taken. Taking water as an instance, we have the following numbers:—

P.	*t* absolute.	Interval of *t*.	P.	*t* absolute.	Interval of *t*.
millim.	°	°	millim.	°	°
10	284·3	27·0	600	366·5	4·2
50	311·3	13·4	700	370·7	3·75
100	324·7	8·4	800	374·45	3·35
150	333·1	6·5	900	377·8	3·05
200	339·6	9·3	1000	380·85	12·35
300	348·9	7·1	1500	393·2	9·3
400	356·0	5·7	2000	402·5	14·0
500	361·7	4·8	3000	416·5	19·35
600	366·5		5000	435·85	

The determinations of the products $\frac{dp}{dt}\,.\,t$ are, however, necessarily less accurate than those of the temperatures, especially at low pressures, for the sources of error are more numerous and have greater influences on the results; yet the values calculated directly from the vapour-pressure curves generally agree very well with those recalculated from the equation

$$R' = R + c\,(t' - t),$$

as will be seen in the following tables. With the exception of chlorobenzene and bromobenzene, the comparison is only made for those substances which have been investigated through a large range of pressure, for otherwise it is impossible to determine the value of c with sufficient certainty.

1. *Water and Carbon Bisulphide.* Ratios of the products $\frac{dp}{dt} . t$ for CS_2 to those of water at the same pressures taken as abscissæ, and the temperatures of water corresponding to those pressures taken as ordinates. $c = -0{\cdot}000399$.

Pressures.	Reduced ratios of $\frac{dp}{dt} . t$, calc. from curves.	Reduced ratios of $\frac{dp}{dt} . t$, recalc. from equation.	Products $\frac{dp}{dt} . t$ for CS_2 calc. from curves.	$\frac{dp}{dt} . t$ for CS_2, recalc. from values for water and recalc. ratios.	Δ per cent.
millim.					
50	·804	·8042	675·7	675·9	+0·03
100	·796	·7989	1280·0	1284·0	+0·31
150	·794	·7955	1839	1842	+0·16
200	·787	·7929	2373	2391	+0·75
300	·787	·7892	3430	3442	+0·35
400	·784	·7864	4436	4451	+0·34
500	·784	·7841	5417	5417	0·0
600	·782	·7822	6361	6364	+0·05
700	·780	·7805	7317	7320	+0·04
800	·783	·7790	8235	8197	−0·46
900	·785	·7777	9281	9196	−0·92
1000	·780	·7764	10102	10054	−0·48
1500	·781	·7715	14507	14325	−1·27
2000	·767	·7678	18460	18481	+0·11
3000	·750	·7622	26056	26476	+1·59
5000	·755	·7545	41128	41106	−0·05

There are only two cases in which the difference between the calculated and recalculated values amounts to 1 per cent., and these two cases occur close together and are in opposite directions. Generally the differences are much smaller than 1 per cent., while the differences between the highest and lowest of the reduced values of $\frac{dp}{dt} . t$ (calculated or recalculated) exceeds 6 per cent.

2. *Bromo-benzene and Chloro-benzene.* The values for these substances appeared to be identical. They were therefore taken together and compared both with carbon bisulphide and with water, to find whether the recalculated values would be the same in the two comparisons. This is shown by the table opposite to be the case.

a. Values of $\frac{dp}{dt}\,.\,t$ for CS_2 taken as unity at each pressure. Temperatures of CS_2 as ordinates. $c = -0{\cdot}0007626$.

b. Values of $\frac{dp}{dt}\,.\,t$ for H_2O taken as unity. Temperatures of H_2O as ordinates. $c = -0{\cdot}0009850$.

Pressures.	$CS_2=1$.		$H_2O=1$.		$\frac{dp}{dt}\,.\,t$ for chloro-benzene. Calculated.	$\frac{dp}{dt}\,.\,t$ for bromo-benzene. Calculated.	Mean values of $\frac{dp}{dt}\,.\,t$ for chloro- and bromo-benzene.	$\frac{dp}{dt}\,.\,t$ for chloro- and bromo-benzene. Recalculated.	
	Mean of reduced ratios of $\frac{dp}{dt}\,.\,t$ for chloro- and bromo-benzene. Calculated.	Reduced ratios of $\frac{dp}{dt}\,.\,t$ for chloro- and bromo-benzene. Recalculated.	Mean of reduced ratios of $\frac{dp}{dt}\,.\,t$ for chloro- and bromo-benzene. Calculated.	Reduced ratios of $\frac{dp}{dt}\,.\,t$ for chloro- and bromo-benzene. Recalculated.				$CS_2=1$.	$H_2O=1$.
millim. 50	1·095	1·0974	0·880	0·8785	...	740	740	741·5	738·4
100	1·086	1·0870	0·8645	0·8653	1384	1395	1390	1391	1391
150	1·068	1·0803	0·848	0·8570	1982	1946	1964	1987	1984
200	1·071	1·0752	0·8425	0·8506	2558	2525	2542	2551	2565
300	1·068	1·0675	0·8405	0·8415	3670	3657	3664	3662	3670
400	1·0625	1·0618	0·833	0·8345	4724	4703	4714	4710	4723
500	1·057	1·0571	0·829	0·8289	5728	5724	5726	5726	5726
600	1·055	1·0531	0·8245	0·8241	6740	6678	6709	6699	6705
700	1·050	1·0496	0·8195	0·8200	7678	7688	7683	7680	7691
800	1·045	1·0465	0·818	0·8163	...	8602	8602	8618	8589

3. *Carbon Tetrachloride and Water.* Values of $\frac{dp}{dt} . t$ for H_2O taken as unity at each pressure. Temperatures of water as ordinates. $c = -0·0006945$.

Pressures.	Reduced ratios of $\frac{dp}{dt} . t$.		$\frac{dp}{dt} . t$ for CCl_4.	
	Calculated.	Recalculated.	Calculated.	Recalculated.
millim.				
50	·859	·8455	722	710·6
100	·830	·8362	1333	1344
150	·827	·8304	1915	1922
200	·832	·8258	2510	2491
300	·811	·8194	3537	3573
400	·809	·8145	4580	4610
500	·823	·8105	5684	5599
600	·819	·8072	6669	6567
700	·794	·8042	7446	7543
800	·799	·8016	8403	8435
900	·789	·7993	9330	9452
1000	·785	·7972	10170	10323
1500	·779	·7886	14470	14642
2000	·780	·7822	18783	18828
3000	·765	·7724	26564	26830
5000	·759	·7590	41344	41351

4. *Ether and Water.* Values of $\frac{dp}{dt} . t$ for H_2O taken as unity at each pressure. Temperatures of water as ordinates. $c = -0·0003194$.

Pressures.	Reduced ratios of $\frac{dp}{dt} . t$.		$\frac{dp}{dt} . t$ for $(C_2H_5)_2O$.	
	Calculated.	Recalculated.	Calculated.	Recalculated.
millim.				
100	·811	·8305	1304	1335
150	·833	·8278	1928	1916
200	·839	·8257	2531	2490
300	·825	·8228	3598	3588
400	·840	·8205	4752	4644
500	·812	·8187	5612	5655
600	826	·8171	6723	6648
700	·818	·8158	7673	7651
800	·811	.8146	8530	8571
900	·808	·8135	9559	9620
1000	·812	·8126	10513	10523
1500	·798	·8086	14819	15013
2000	·822	·8057	19787	19393
3000	·798	·8012	27730	27830
5000	·795	7950	43314	43313

5. *Chloroform and Water.* Values of $\frac{dp}{dt} . t$ for water taken as unity at each pressure. Temperatures of water as ordinates. $c = -0{\cdot}0006701$.

Pressures.	Reduced ratios of $\frac{dp}{dt} . t$.		$\frac{dp}{dt} . t$ for CCl_3H.	
	Calculated.	Recalculated.	Calculated.	Recalculated.
millim.				
200	·854	·8545	2575	2577
300	·836	·8483	3646	3699
400	·843	·8436	4772	4775
500	·844	·8397	5829	5801
600	·842	·8365	6850	6806
700	·838	·8337	7862	7819
800	·838	·8311	8812	8745
900	·821	·8289	9709	9802
1000	·838	·8269	10846	10708
1500	·818	·8186	15191	15199
2000	·805	·8123	19364	19552
3000	·801	·8030	27813	27893
5000	·777	·7900	42320	43041

6. *Ethyl Bromide and Ethyl Chloride with Water.* Values of $\frac{dp}{dt} . t$ for water taken as unity at each pressure, and compared with mean values of $\frac{dp}{dt} . t$ for ethyl bromide and ethyl chloride. Temperatures of water as ordinates. $c = -0{\cdot}0006335$.

Pressures.	Mean of reduced ratios of $\frac{dp}{dt} : t$.		Mean values of $\frac{dp}{dt} . t$ for C_2H_5Br and C_2H_5Cl.	
	Calculated.	Recalculated.	Calculated.	Recalculated.
millim.				
50	·851	·8449	715	710·1
100	·849	·8364	1364	1344
150	·829	·8311	1919	1924
200	·823	·8271	2483	2495
300	·8085	·8211	3525	3581
400	·8045	·8166	4555	4622
500	·8085	·8130	5587	5616
600	·817	·8099	6647	6589
700	·8085	·8073	7580	7572
800	·8045	·8049	8463	8469
900	·7955	·8028	9407	9493
1000	·798	·8008	10330	10370
1500	·793	·7930	14721	14723
2000	·790	·7871	18994	18946
3000	·7835	·7783	27207	27035
5000	·7615	·7660	41504	41733

7. *Mercury and Water.* From determinations of vapour-pressures of mercury by Ramsay and Young *. If Regnault's values are correct, the equation $R' = R + c(t' - t)$ does not hold good. Values of $\frac{dp}{dt} . t$ for water taken as unity at each pressure. Temperatures of water as ordinates. $c = -0·0004236$.

Pressures.	Reduced ratios of $\frac{dp}{dt} . t$.		$\frac{dp}{dt} . t$ for Mercury.	
	Calculated.	Recalculated.	Calculated.	Recalculated.
millim. 10	·866	·8600	160	158·9
50	·850	·8486	714	713·2
100	·833	·8429	1339	1355
150	·843	·8393	1950	1943
200	·835	·8366	2519	2523
300	·834	·8326	3639	3631
400	·827	·8296	4682	4696
500	·827	·8272	5715	5714
600	·829	·8252	6741	6714
700	·824	·8234	7731	7723
800	·829	·8218	8717	8647
900	·814	·8204	9621	9701
1000	·818	·8191	10595	10607
1500	·811	·8139	15064	15112
2000	·816	·8099	19633	19494
3000	·804	·8040	27919	27928
5000	·800	·7958	43562	43356

8. *Mercury and Carbon Bisulphide.* Employing the values of $\frac{dp}{dt} . t$ for mercury and carbon bisulphide, both recalculated from their ratios with those of water, these values bear an almost constant ratio to each other at all pressures, or $c = 0$. The calculated ratios vary only between 1·0547 and 1·0552.

XXXVI. *The Winding of Voltmeters.* *By* Professors W. E. Ayrton, *F.R.S., and* John Perry, *F.R.S.*†

I. The use of voltmeters for measuring the potential-difference existing between the mains in electric light circuits, and for enabling us to maintain this potential-difference constant, is so wide in its possible application, that it is unnecessary to dwell on the importance of voltmeters giving accurate indications. Indeed, the sensibility of an incandescent lamp is so great (the light, as we have shown in a previous communication to this Society*, being in many cases proportional to the cube of the potential-difference minus a constant), that it is necessary that voltmeters should record with great accuracy the potential-difference, as well as quickly indicate any changes that from time to time take place in this potential-difference.

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* The data for these determinations are to be found in the 'Journal of the Chemical Society,' January 1886.

† Read June 13, 1885.

It is therefore desirable to consider what are the most important errors in commercial voltmeters as at present constructed, and how these errors may be practically diminished or altogether removed. They are:—

1. An error arising from the sensibility of the voltmeter varying with its resistance, and therefore with its temperature. This change of temperature is due partly to the variation of the temperature of the room and partly to the coils of the instrument becoming heated by the passage of the current through them.

2. An error arising from the sensibility of the instrument being temporarily varied by external magnetic disturbance.

3. An error arising from certain types of voltmeters altering in sensibility so that their constants vary from time to time.

4. An error arising from the volt standards employed by different makers of voltmeters differing several per cent. from one another.

The variation in the resistance of a voltmeter produced by changes of temperature of the room can of course be made very small by winding the instrument with German-silver wire, or better with platinum-silver, or, best of all, with wire made of platinoid, or it may be made absolutely nought by use of an outside resistance of carbon; or, lastly, if the voltmeter be simply wound with copper wire, this error, although large, can be accurately allowed for if the temperature of the voltmeter be known. But the error arising from the heating of the voltmeter by the passage of the current through it is much more difficult to allow for, as this rise of temperature of the instrument will be unknown. Hence it is important to reduce this error to a minimum, and it does not at all follow that the employment of German-silver wire or even platinoid wire will effect this result; indeed we have already given, in a previous communication made to this Society*, general reasons showing that German silver is undesirable to be used in the winding of voltmeters. When, however, the diameter of the coil of the instrument exceeds a certain limit, it would appear that German-silver wire might be better for the outer coils: hence the following investigation has been made for the purpose of ascertaining whether in an instrument of the size of our Solenoid, or Long-Range Voltmeter, any portion of the coil should be wound with German-silver wire, and if so, how large a portion, also what should be the law of variation of sectional area of the copper and German-silver wire with the diameter of the convolution in which it is wound, so that the heating-error produced by the passage of the current through the voltmeter shall be a minimum.

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* "The most Economical Potential-Difference to employ with Incandescent Lamps," Proc. Phys. Soc. vol. vii. p. 40 [Phil. Mag. [5] vol. xix. p. 304 (April 1885)].

Although the dimensions of the instrument adopted in the following investigation are those of this special form of voltmeter, the mode of considering the problem will apply equally well to any other galvanometer used as a voltmeter, or to the fine wire coil of an Ohm-meter, Powermeter, and Ergmeter, or indeed in any case where it is desired to produce a given magnetic effect with a minimum error due to heating. It is only necessary to consider the case of a cylindrical coil of internal radius r_0 and external r_1 and length l, as it is easy to extend our reasoning to a coil of any other shape and dimensions.

II. At a place in the coil at a distance r from the axis let the cross section of the wire be x, let ρ be the specific resistance of the material, and γ the rate at which ρ increases with temperature; x, ρ, and γ being functions of r. For simplicity of calculation we shall neglect the volume of the insulating material. Let one spire of radius r produce a magnetic effect of the amount

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* "Direct-reading Electro-Measuring Instruments, and a Non-Sparking Key," Proc. Phys. Soc. vol. vi. p. 69 [Phil. Mag. [5] vol. xvii. p. 304 (April 1884)].

$$KCr^d, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

C being the current, and d having in almost all instruments a value which lies between the limits -1 and 0. Also let

$$x=x_0 r^a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\rho=\rho_0 r^b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\rho\gamma=\rho_0\gamma_0 r^c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In a layer of wire of thickness δr there are $\frac{l\,\delta r}{x}$ spires of wire, and the resistance of the layer is

$$\frac{2\pi r\rho l}{x^2}\delta r, \quad \text{or} \quad \frac{2\pi r\rho_0 r^b l}{x_0{}^2 r^{2a}}\delta r, \quad \text{or} \quad \frac{2\pi\rho_0 l r^{1+b-2a}}{x_0^2}\delta r;$$

so that the whole resistance of the coil is evidently

$$R=\frac{2\pi\rho_0 l}{m x_0^2}(r_1{}^m-r_0{}^m) \text{ if } m=2+b-2a. \quad . \quad . \quad (5)$$

The magnetic effect of the spires in the thickness δr is

$$KCr^d l\frac{\delta r}{x} \quad \text{or} \quad \frac{KCl}{x_0}r^{d-a}\delta r,$$

so that the whole magnetic effect of the coil is

$$M=\frac{KCl}{x_0 n}(r_1{}^n-r_0{}^n) \text{ if } n=d-a+1. \quad . \quad . \quad (6)$$

We shall now assume that we can, by introducing thin strips of copper among the windings, give the same ease of cooling to unit volume of the wire everywhere; that is, after the current has flowed for some time, that the increase of temperature at any point is proportional to the rate at which current heat is produced per unit volume. Now

$$C^2\frac{2\pi\rho_0}{x_0^2}r^{1+b-2a}\,l\delta r \quad \text{or } C^2\frac{\rho_0}{x_0^2}r^{b-2a}\,.\,2\pi r l\delta r$$

is the rate at which heat is produced in a layer of thickness δr.

This rate is $\frac{\rho_0}{x_0^2}r^{b-2a}$ times the volume of the layer. Therefore the rate at which heat is produced *per unit volume* is $C^2\frac{\rho_0}{x_0^2}r^{b-2a}$: therefore, by hypothesis, the increase of temperature is proportional to $C^2\frac{\rho_0}{x_0^2}r^{b-2a}$.

The increase of resistance of this layer is the resistance of the layer multiplied by γ times the increase of temperature, and is therefore proportional to

$$\frac{2\pi\rho_0 l r^{1+b-2a}}{x_0^2}\delta r \,.\, \gamma \,.\, \frac{C^2\rho_0}{x_0^2} r^{b-2a},$$

or to

$$\gamma \frac{C^2\rho_0^2}{x_0^4} r^{1+2b-4a},$$

where $\gamma=\gamma_0 r^{c-b}$, and the whole increase of resistance is proportional to

$$\int_{r_0}^{r_1} \gamma_0 C^2 \frac{\rho_0^2}{x_0^4} r^{1+b+c-4a}\, dr,$$

or

$$\gamma_0 \frac{C^2\rho_0^2}{p x_0^4}(r_1^p - r_0^p), \text{ if } p=2+b-4a+c; \quad . \quad . \quad (7)$$

so that the fractional change of resistance is proportional to

$$\frac{\gamma_0 C^2 \rho_0^2}{R x_0^4 p}(r_1^p - r_0^p). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Now regarding M as constant, and substituting for C and R from (6) (5) and (8) we observe that the fractional change of resistance due to heating is proportional to

$$\gamma_0\rho_0 \cdot \frac{n^2}{(r_1^n - r_0^n)^2} \cdot \frac{m}{r_1^m - r_0^m} \cdot \frac{r_1^p - r_0^p}{p} \quad . \quad . \quad . \quad (9)$$

and it is required to find the values of a, b, and c for which this expression is a minimum.

III. It may be seen that the error (9) is less as $\gamma_0\rho_0$ becomes less, therefore $\gamma_0\rho_0$ should be made as small as possible so that *when wires of two metals, as for example copper and German silver, are employed, the wire for which $\gamma\rho$ has the least value, that is the copper, should be used for the inner winding.* It is, however, when we consider the waste of electric energy in the coil, that the great objection to the use of German silver for the internal winding becomes still more striking. For the rate of production of heat in the coil is proportional to C^2R, which, from (6) and (5), equals

$$\frac{x_0^2 n^2 M^2}{K^2 l^2 (r_1^n - r_0^n)} \times \frac{2\pi\rho_0 l}{m x_0}(r_1^m - r_0^m).$$

Hence to produce the same magnetic effect M in a coil of given length l, the rate of production of heat is proportional to

$$\frac{n^2}{(r_1{}^n-r_0{}^n)^2}\times\frac{r_1{}^m-r_0{}^m}{m}\rho_0. \quad . \quad . \quad . \quad . \quad (10)$$

Consequently when a given angular deflection is produced, the rate of wasting electrical energy in heat will be thirteen times as great if German-silver wire be used as it will be if copper is used, no matter what be the gauge of wire employed, that is, whether the voltmeter be wound to measure a maximum potential-difference of 50 or of 500 volts.

Eliminating r from (3) and (4) it follows that

$$\left(\frac{\gamma\rho}{\gamma_0\rho_0}\right)^{\frac{1}{c}}=\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{b}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where b may have any arbitrary value. Hence if there is a regular change possible in the value of $\gamma\rho$ such as given in (4), we must have

$$c=\mathrm{P}b,$$

where P is some constant. Equation (11), therefore, becomes

$$\frac{\gamma\rho}{\gamma_0\rho_0}=\left(\frac{\rho}{\rho_0}\right)^{\mathrm{P}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If, therefore, our wire in the inside of the coil is of copper and on the outside of German silver,

$$\gamma\rho=\frac{13}{9}\gamma_0\rho_0,$$

or

$$\rho=13\,\rho_0.$$

Hence from (11)

$$\frac{13}{9}=13^{\mathrm{P}},$$

or

$$\mathrm{P}=0{\cdot}143,$$

and

$$c=0{\cdot}143\,b. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

It is certain that generally the relationship between γ and ρ cannot be of the simple character here assumed; but inasmuch as our results are only applied in cases where the windings are either of German silver or of copper, or of the two together, and in these cases the expressions used are quite correct, there is no practical objection to the use of such a law. The law is very nearly the same as the assumption that a wire of any material, lying electrically between German silver and copper, is made up of wires of German silver and copper.

Since from (7),

$$p=2+b-4a+c,$$

it follows, using (13), that

$$p=2+1{\cdot}143b-4a, \quad \ldots \quad (14)$$

and the values of m and n are given respectively in terms of a, b, and d in (5) and (6), and are

$$m=2+b-2a \quad \ldots \quad (14)$$

$$n=d-a+1. \quad \ldots \quad (14)$$

In our Solenoid Voltmeter, r_0 equals 1, r_1 equals 8, and d equals -1; and since the greatest value of ρ is $13\rho_0$, it follows that r^b cannot be greater than 13, but b is a constant, and the greatest value of r is 8, therefore b must be less than 2. From (14), since b is always small, we see, after a has reached a certain large value, that p, m, and n increase numerically as a increases, and are positive when a is negative, and *vice versa*; also that, finally, p equals $2m$ or $4n$.

Substituting the values for r_0 and r_1, the error (9) may be written

$$\frac{\gamma_0\rho_0\left(\frac{n}{8^n-1}\right)^2\left(\frac{m}{8^m-1}\right)}{\frac{p}{8^p-1}},$$

and it may be seen by inspection that each of the factors of which it is composed is always positive, whether m, n, and p be positive or negative. Therefore the error is always positive. For large values of m, n, and p, the error may be written

$$\frac{\left(\frac{n}{8^n-1}\right)\left(\frac{2n}{8^{2n}-1}\right)}{\frac{4n}{8^{4n}-1}},$$

which equals

$$\frac{\left(\frac{n}{8^n-1}\right)^2}{\frac{2}{8^{2n}+1}},$$

or

$$\frac{n^2({}^{2n}+1)}{8^{2n}+1-2\times 8^n}.$$

This expression is always greater than n^2 whether n be positive or negative, and is therefore very great since n is great. Consequently for large positive or negative values of m, n, and p, the error (9) becomes very great, and therefore it can have no maximum. The values of a and b, therefore, found from equating to nought its differential coefficients with respect to a and to b, will give a minimum value of this error. And the error will be a minimum when its logarithm is a minimum. Now taking its logarithm, putting r_0 equal to 1, and r_1 equal to 8, differentiating with regard to a and b respectively, and equating to zero, we arrive at the equations

$$\phi(n)-0{\cdot}857\,\phi(p)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

$$\phi(m)-1{\cdot}143\phi(p)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

where

$$\phi(a)=\frac{\log_e 8}{1-8^{-a}}-\frac{1}{a}. \quad . \quad . \qquad (17)$$

To assist in the solution of these simultaneous equations we calculated the following Table:—

TABLE I.

a.	$\phi(a)$.
− 3	0·3293
− 2	0·4670
− 1	0·7029
0	1·0397
1	1·3764
2	1·6124
3	1·7502
4·5	1·8474
10	1·9794
100	2·0694

And in order to solve these equations we used the following artifice:—A curve was first plotted on squared paper with $\phi(a)$ as ordinate and a as abscissa, and by means of this curve for any value of $\phi(a)$ given we could at once find a approximately.

Table II. was then arrived at in the following way:— value of p was chosen, and $\phi(p)$ written down. Then since $0{\cdot}857\,\phi(p)$ is equal to $\phi(n)$ by (15), n was known from this by the curve for $\phi(a)$; m was next found in the same way from (16). Thus columns 1 and 4 in Table II. contain values of p and n which satisfy (15), and which were obtained in this way, while in columns 1 and 6 we have values of p and m which satisfy (16).

TABLE II.

p.	$\phi(p)$.	$0{\cdot}857\,\phi(p)$, or $\phi(n)$.	n.	$1{\cdot}143\,\phi(p)$, or $\phi(m)$.	m.
2	1·6124	1·381	1·03	1·842	4·35
1	1·3764	1·18	0·39	1·572	1·78
0	1·0397	0·8913	−0·425	1·189	0·425
−1	0·70295	0·6026	−1·370	0·8035	−0·69
−2	0·467	0·4002	−2·38	0·5337	−1·69

Now first using the pair of columns 1 and 4, we chose corresponding values of p and n, say 2 and 1·03; then, since from (14)

$$2=2+1{\cdot}143b-4a,$$

and

$$1{\cdot}03=d-a+1,$$

we could calculate a pair of values for a and b which satisfy (14) and (15), since d was assumed to be known. Next taking another pair of values of p and n from Table II., another pair of values of a and b was calculated, and so on. Plotting these values of a and b as coordinates of points on squared paper we obtained the curve *s s s*, fig. 1 *, the coordinates of every point of which are the values of a and b that satisfy (15). In the same way we drew the curve *t t t* for (16), and the coordinates of the point of intersection Z are the values of a and b which satisfy (15) and (16), and therefore make the error (9) a minimum.

Thus for example, taking $d=-1$, as seems to be the case in our solenoid instrument, we find that, with some approximation to accuracy,

$$\mathbf{a}=0{\cdot}19 \text{ and } \mathbf{b}=-0{\cdot}82$$

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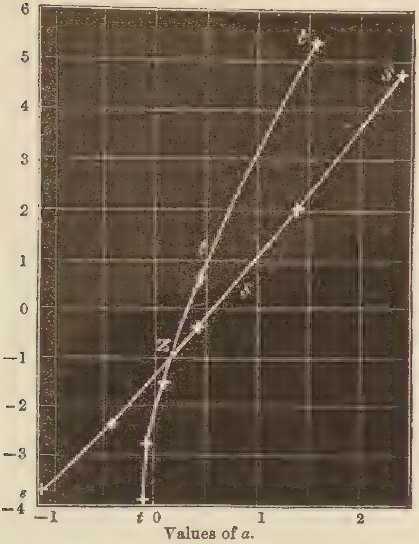
* This and the two succeeding figures are only about one twelfth of the size each way of the figures actually employed in the calculations.

Fig. 1.



for minimum error. But if we have used our material of least resistance for the inner winding, b cannot in practice be less than 0, for ρ cannot be less than ρ_0, and since r is never less than 1, ρ would be some fraction of ρ_0 if b were less than 0. Hence in practice we find that b must equal 0, that is, *a less heating-error is produced when all the wire is of copper than when we use German-silver windings on the outside of the coil.*

IV. Making b equal to nought as the least value of b attainable, we can now find the value of a to make the error a minimum. To do this we must put b equal to nought, take d equal to -1 in (9), differentiate with respect to a, and equate the result to 0.

Thus $$\phi(m)+\phi(n)-2\phi(p)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $\phi(a)$ has the signification given to it in (17).

Since $$b=0$$
and $$d=-1,$$
it follows from (14) that

$$\left.\begin{aligned} m&=2-2a \\ n&=-a \\ p&=2-4a, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

therefore, from (17)

$$\phi(2-2a)+\phi(-a)-2\phi(2-4a)=0.$$

This equation cannot be solved generally, but by giving various values to a we can find which of these values makes the expression on the left-hand side most nearly equal to nought, and to ascertain the value of the function for each of the values of a we can conveniently use the curve connecting $\phi(\alpha)$ and α previously employed.

For $a=0{\cdot}5$ we have

$$\phi(1)+\phi(-0{\cdot}5)-2\phi(0)=1{\cdot}375+0{\cdot}865-2\times1{\cdot}04$$
$$=+0{\cdot}16\,;$$

for $a=0{\cdot}4$ we have

$$\phi(1{\cdot}2)+\phi(-0{\cdot}4)-2\phi(0{\cdot}4)=1{\cdot}43+0{\cdot}9-2\times1{\cdot}182$$
$$=-0{\cdot}034\,;$$

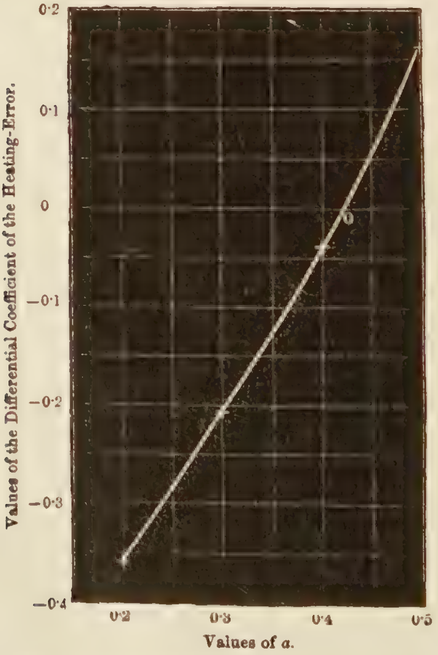
for $a=0{\cdot}3$ we have

$$\phi(1{\cdot}4)+\phi(-0{\cdot}3)-2\phi(0{\cdot}8)=1{\cdot}48+0{\cdot}936-2\times1{\cdot}311$$
$$=-0{\cdot}206\,;$$

for $a=0{\cdot}2$ we have

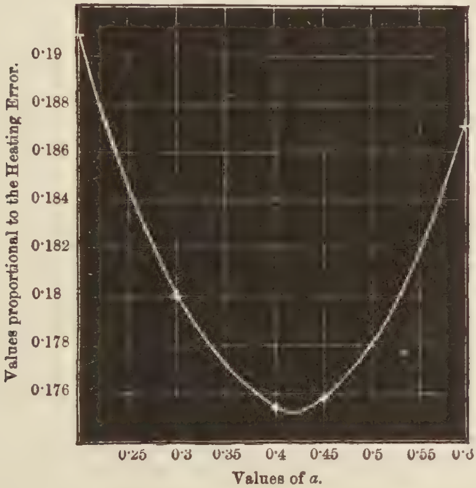
$$\phi(1{\cdot}6)+\phi(-0{\cdot}2)-2\phi(1{\cdot}2)=1{\cdot}53+0{\cdot}97-2\times1{\cdot}43$$
$$=-0{\cdot}36.$$

Fig. 2.



Plotting these points, we find (fig. 2) that the curve for the differential coefficient cuts the axis along which a is measured at a point 0, for which a has the value 0·42; or, in other words, a *equal to* 0·42 *makes the differential coefficient equal to nought, and hence is the value of* a *that makes the heating-error a minimum.* As a check on this we have drawn the curve for the values of the heating-error for different values of a (fig. 3), and from this also it is seen that the heating-error has a minimum when a equals 0·42. Hence it follows that the law of increase of the sectional area of the wire with the radius of the convolutions should in our Solenoid Voltmeter follow the law

Fig. 3.



$$x = x_0 r^{0\cdot42},$$

in order that the error due to heating may be a minimum.

Fig. 3 not only gives the value of a that makes the heating-error a minimum, but it shows by how much the heating-error is increased if a has a value greater or less than 0·42. For example, if a equals 0, that is to say if the voltmeter be all wound with wire of the same gauge, then, continuing the curve backwards, we find that the heating-error is increased by about one third.

V. One plan of diminishing the heating-error with voltmeters, and one that we have frequently employed, is to place in the same circuit with the instrument a coil wound with German-silver wire, and having a resistance far greater than that of the voltmeter itself. By using wire of large gauge, wound on a bobbin of large volume, and with as efficient an arrangement for cooling as can be conveniently employed, the heating-error can be rendered extremely small. Such a bobbin, however, is both costly and bulky, and the employment of such an outside resistance-coil wastes energy, and can only be used with voltmeters in electric light and power installations where such a waste of energy is unimportant. The following investigation has therefore been made for the purpose of determining what must be the volume of this outside resistance-coil, and what must be its resistance relatively to that of the voltmeter-coil, so that the heating-error may not exceed any given value.

Let R be the resistance of the coil of copper wire wound on a voltmeter, let H be the rate of production of heat when the maximum deflection is obtained; then, as already shown, H will be constant for a voltmeter of a given size wound according to a definite law, whether it be wound with fine wire for a 500-volt instrument or with thicker wire for a 50-volt one. Let C be the current passing round it when this maximum deflection is being produced; then

$$C^2R = H, \text{ a constant.}$$

Let the increase of resistance produced by the passage of the current for some time be r; then $\frac{r}{R}$ is proportional to the product of γ, the coefficient of increase of the resistance of copper per degree of temperature, into H, since, as already explained, the voltmeter is so wound that the rate of production of heat is proportional to the increase of temperature. Hence

$$\frac{R}{r} = \text{a constant, A say.}$$

Now let there be a resistance-coil of German-silver wire of uniform cross section, of volume V in series with the instrument, and let us suppose that for any coils thus added the rise of temperature is proportional to the energy wasted per second per unit volume, that is to say proportional to the rate of production of heat per unit volume.

Let R_1 be the resistance of this added coil, then $\frac{C^2R_1}{V}$ is proportional to its rise of temperature; hence, if r_1 be the increase of resistance and γ_1 the coefficient of increase of resistance of German silver per degree,

$$\frac{r_1}{R_1} \propto \gamma_1 \frac{C^2R_1}{V},$$

or

$$r_1 = B\frac{C^2R_1^2}{V} \text{ say,}$$

where B is a constant. Hence, if e is the error due to the proportional change of resistance,

$$e = \frac{r+r_1}{R+R_1},$$

$$= \frac{AR+\frac{BC^2R_2}{V}}{R+R_1}.$$

Let

$$\frac{R_1}{R} = y,$$

then

$$e = \frac{A+\frac{B}{V}{}^2y}{1+y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

To determine the value of the constants A and B we may proceed as follows:—Wind a voltmeter according to the law which is to be adopted with any or no resistance-coil outside; find the potential-differences P_1 and P_2 required to produce the maximum reading of the instrument,

1st, immediately on making connection;

2nd, after connection has been made for some time; then

$$e = \frac{P_2 - P_1}{P_1}.$$

Let e be experimentally determined in two cases,—one where there is no resistance-coil outside, and the other where there is such an auxiliary resistance-coil, obtaining values e_1 and e_2 respectively. Measure y, that is the ratio of R_1 to R, and also V. Then from the equations

$$e_1 = A,$$

$$e_2 = \frac{A+\frac{B}{V}y^2}{1+y},$$

the values of the constants A and B can be determined; and when determined, the error arising with the employment of an outside resistance-coil of any resistance and of any volume can be calculated from (20).

It is obvious that as V becomes greater the error becomes less and less, but there is a practical inconvenience in largely adding to the weight of the instrument.

VI. *To determine the best resistance to give to the outside coil of given volume.*—When V is fixed, e is a maximum or minimum, when

$$\frac{de}{dy}=0.$$

Now

$$\frac{de}{dy}=\frac{\frac{B}{V}y^2+2\frac{B}{V}y-A}{(1+y)^2},$$

and

$$\frac{d^2e}{dy^2}=\frac{2\left(\frac{B}{V}+A\right)}{(1+y)^3},$$

y must be positive, therefore $(1+y)^3$ is positive, also A and B are both positive, therefore $\frac{d^2e}{dy^2}$ is positive, and therefore e is a minimum, when

$$\frac{de}{dy}=0,$$

that is, when

$$\frac{B}{V}y^2+2\frac{B}{V}y-A=0,$$

or

$$y=-1\pm\sqrt{1+\frac{AV}{B}};$$

but y cannot be negative, therefore only the positive sign can be used, or

$$y=\sqrt{1+\frac{AV}{B}}-1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

makes e a minimum.

Substituting this value of y in (20), we find that

$$e=2\frac{A\quad\frac{B}{V}-\sqrt{1+\frac{AV}{B}}}{\sqrt{1+\frac{AV}{B}}} \quad . \quad . \quad . \quad . \quad (22)$$

VII. *To determine the best volume, and the best resistance to give to the outside coil for a given permissible error.*—From the preceding it is easy to see that if the maximum allowable value of e is given, the minimum value of the volume that can be employed is given from (22), and for this vaue of the volume, the best value of y, that is the best ratio of the resistance of the outside coil to that of the magnetizing coil of the voltmeter, can be calculated from (21).

VIII. *To determine the sectional area to be given to the inner wire of a voltmeter in terms of the maximum potential difference so as to make the heating-error a minimum.*—By experiment ascertain the potential difference that is necessary to produce the maximum deflection with any voltmeter of the shape and size to be employed and wound according to the law

$$x = x_0 r^a,$$

where a has the value that makes the heating-error a minimum, and which value will depend on the shape and size of the instrument, and may be calculated for any special type of voltmeter in the same way as it has been calculated in what precedes for the Long-Range Magnifying Spring-instrument. Then if it is desired to wind another instrument, of the same volume and shape, so that it will have its maximum deflection given by P_2 volts, it follows, since the heating will, as already proved, be the same in the two instruments for their maximum deflections, that their resistances must be proportional to the squares of P_1 and P_2, and hence, from (5), since the same law of winding is followed, that the cross section of the innermost layer of wire in the two instruments must be inversely proportional to P_1 and P_2.

To have the same heating-error in the two cases, with outside resistance-coils of the same shape and size, it follows from (21) that it is necessary to use the same ratio between the resistance of the outside coil and the resistance of the magnetizing coil. To have a less or greater heating-error in the second than in the first case for the maximum deflection, it is only necessary to determine from (22) (the constants of which have been deduced by experiment with the first instrument) the greater or less volume V of the outside resistance-coil needed with the second instrument, and calculate the ratio y of the resistance of the outside coil to that of the magnetizing coil by (21).

IX. As to the second error, arising from the sensibility of the instrument being temporarily varied by external magnetic disturbance referred to in § I. This can be overcome in three distinct ways: by

1. Shielding the instrument by putting it in an iron case with very thick walls.
2. Using a very powerful controlling field.
3. Using a motion of *translation*, and *not of rotation*, for the body in the instrument that is deflected by the current.

The first method to be successful requires that the iron case should be heavy and cumbersome, and makes it unsuitable for portable instruments.

A very powerful controlling field necessitates a very powerful deflecting force, and so either makes the instrument unsensitive or necessitates a considerable expenditure of electric energy in the instrument. The third method, consisting in giving the deflected body a *motion of translation*, is the best, since, no matter how strong the disturbing magnetic field may be, provided it is uniform throughout the small space traversed by the deflected body on its motion, no increase nor diminution of the deflection will be caused by the outside magnetic field. For this reason we have adopted this third method in our Magnifying Spring-instruments, and experiment shows that the shielding produced in this way is even better than in our older form of instruments provided with a powerful permanent magnet.

X. As to the third error, arising from certain types of voltmeters altering in sensibility so that their constants vary from time to time. This error arises from two causes. In some voltmeters, such as those in which a controlling permanent magnet is employed, there is an error which gradually comes on from the demagnetization of this magnet, an error which is greatly increased by a mistaken careful regular employment of a keeper for the magnet when the instrument is not in use. This error can be avoided by dispensing with a permanent magnet, and using a spring or a weight, or an electromagnet which is magnetized by the current to be measured. A second time-error arises from the employment of iron in the neighbourhood, and has the effect of making the indications of the instrument depend not merely on the current passing at the time of measurement, but also on the currents that have lately passed through the instrument. This error is especially marked in those instruments in which the controlling force, or the force tending to bring the needle back to zero in opposition to the deflecting force, is produced by the attraction of an electromagnet containing a core of soft iron wire, and which it is *assumed* is magnetized to saturation by the current which it is our object to measure. With small currents, however, which produce deflections in the lower part of the scale, this iron core is not saturated, and differences in the values of the readings of as much as 30 to 40 per cent. are obtained, depending on whether it is an increasing or a decreasing current that is being measured with such an electromagnetic instrument.

XI. The fourth error, arising from the volt standards employed by various people differing several per cent. from one another, has been hitherto a very serious matter The standards employed have usually been—

1. The E.M.F. of a Daniell's cell.
2. The E.M.F. of a Latimer Clark's cell.
3. A standard depending on the electrochemical equivalent of silver and the value of the legal ohm.
4. The indication of a voltmeter graduated by some well-known maker, but probably graduated in a way quite unknown to the user.

Standard 1 is vague unless the details of the method employed in setting up the Daniell's cell are more carefully specified than heretofore; for example, Carpart, in some recent tests of the E.M.F. of a Daniell's cell, made out of a U-tube, with a contraction in the lower part, found it to vary from 1·111 legal volts when the zinc-sulphate solution had 25 per cent. of $ZnSO_4$, to 1·142 legal volts when the solution had only 3 per cent. of $ZnSO_4$. His method of testing is based on the electrochemical equivalent of silver and the legal ohm, and as a proof of its accuracy he finds that the E.M.F. of a Latimer Clark's cell is 1·434 volts. If, however, we compare his results for a Daniell's cell with 1·072 volts as found by Sir W. Thomson when the zinc-sulphate solution had a specific gravity of 1·2, we see how Daniell's cells differ among one another*.

The E.M.F. of a Latimer Clark's cell has also some vagueness attached to it, unless the temperature-coefficient of variation of E.M.F. of the particular Latimer Clark's cell is known and taken account of. Generally the users of these cells are contented to say that the E.M.F. of a Latimer Clark's cell, as determined by Lord Rayleigh, is 1·435 true volts, or 1·438 legal volts, and they neglect the temperature variation altogether, which amounts to about 1 per cent. diminution per 12° C. elevation of temperature with certain Latimer Clark's cells.

Standard No. 3 is the one that we have used for some time; and as we have obtained very satisfactory results with it, it may be well to describe our mode of using it a little in detail. First, by means of a large number of direct comparisons with a copper voltameter, we determine the most probable value in amperes of the indications in different parts of the scale of an ammeter *fixed permanently in position* by being screwed to the table. A current is now sent through this ammeter in series with a long thick copper wire having about 6 ohms resistance and wound in a large open coil. This coil has attached to its two ends the standard voltmeter, which is also fixed *permanently in position*. Simultaneous readings of the ammeter and voltmeter are now taken by two observers, when a signal is given and the bridge-piece of a mercury commutator is moved so as to cut off the current and connect the copper wire together with the voltmeter, which is joined with it in parallel, to a differential galvanometer. Then *time* measurements, for about ten minutes, are taken by a third observer of the parallel resistance of the thick copper wire and the attached voltmeter. Next a curve is drawn connecting the resistance of the cooling coil with the time, and on completing the curve backwards we ascertain with great accuracy, by interpolation, the resistance at the moment of stopping the current, that is at the moment of making the simultaneous observations on the ammeter and voltmeter. Our copper wire is sufficiently thick that for a current of some 15 amperes passing through it, corresponding with about 80 volts maintained at its ends, the time-curve of its resistance, obtained after stopping the main current, is nearly a straight line with hardly any inclination to the axis along which the time is measured ; or, in other words, the change of resistance due to cooling while the measurements of resistance are being made is very small and easily allowed for. And this resistance, in legal ohms, multiplied by the current passing in amperes, gives of course the potential difference in legal volts.

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* Since the reading of this paper this subject has been very fully treated by Dr. Fleming, in a paper "On the Use of Daniell's Cell as a Standard of Electromotive Force," read before the Physical Society on June 27, 1885 [Phil. Mag. [5] vol. xx. p. 126 (August 1885)].

In using the copper voltameter to calibrate the ammeter it is of course necessary, in order that a firmly adherent deposit should be obtained, that the area of the immersed surfaces of the anode and cathode opposed to one another should exceed the limit given by Dr. Hammerl, viz. one square decimetre per 7 amperes, or rather more than two square inches per ampere ; and in our experiments we have taken care to use a larger area in proportion to the current. Our negative plates consisted of a number of separate sheets of very thin hard copper, so that the weight of the deposit could be accurately measured ; the current was supplied by accumulators and increased or diminished by a carbon resistance worked with a screw. By watching the ammeter and slightly turning this screw one way or the other, the current could be kept very constant for a long time. The value of the electrochemical equivalent employed for copper was 0·3295 milligrammes per coulomb, which is deduced from the mean of the electrochemical equivalents of silver, 1·118 milligrammes per coulomb as given by Lord Rayleigh, and 1·1183 by F. and W. Kohlrausch, and by using 63·47, 107·56, 32·07, and 16 as the atomic weights of copper, silver, sulphur, and oxygen respectively. The solution of copper sulphate employed was a saturated one, the crystals of copper sulphate used in making it being chemically pure.

To avoid errors being introduced by oxidation of the copper that had just been deposited on the copper plates, they were, on being taken out of the sulphate of copper bath, on the stoppage of the current, immediately and quickly washed, first in water, secondly in alcohol, thirdly with ether poured over them, and then dried.

The current that a given set of accumulators will send through a copper voltameter depends of course partly on the distance between the alternate positive and negative plates, and partly on the depths to which they are immersed in the saturated solution of copper sulphate. The proper arrangements of the plates to allow any particular current to pass cannot conveniently be determined with the plates that are going to be used for the experiment, as an unknown weight of copper would be deposited on them during the process of adjustment, and great delay would be introduced if the plates had to be dried and weighed after the adjustment and before making the final experiment. Hence after the proper plates to be used as the anodes had been very carefully cleaned and weighed, a spare set of exactly the same size and shape were put into the bath, and their distance, as well as the carbon resistance, adjusted until the desired current was passing; then the spare set was removed and the weighed set substituted for them, and the experiment carried out.

Having determined our volt-standard in the way above explained, we tested various voltmeters of different makers, and we found that whereas the indications of some of them agreed very well with our standard, the indications of others differed by several per cent.

XXXVII. *On a Machine for Solving Equations.*

By C. V. BOYS*.

[Plate V.]

AT a late meeting of the Physical Society Mr. C. H. Hinton showed some apparatus that he had made to explain the nature of equations. One of the instruments for representing the case of a simple equation was manageable enough; but the second, which dealt with the quadratic, could hardly be employed to solve a quadratic, as the root was not capable of continuous variation. It could, therefore, only after considerable trouble in trials and errors had been taken, find those real roots which consisted of whole numbers between the limits of the machine, say + and −10.

At the time that Mr. Hinton was trying to devise a quadratic machine (with the object not so much of solving quadratic equations, as of assisting boys to understand the method employed, now about ten years ago), he showed me what he had done; and it then occurred to me that continuous variation might be employed in a machine so that fractional roots could be found, and that the degree need not be limited to a quadratic, but that a cubic or one of any power of the form $a+bx+cx^2+dx^3+\&c.=0$ could be solved. I then half completed the machine I have now the honour of exhibiting before the Physical Society, but laid it on one side till the present time.

I find that Mr. A. B. Kempe has invented and described† a machine with the same object. He replaces the expression x by $\cos\theta$, and then by a known process changes the equation in which the terms are powers of cosines of θ into one in which the terms are cosines of multiples of θ. The equation is then in a state to be dealt with by his machine. A series of levers jointed on one another, the first hinging on the intersection of a pair of rectangular axes, have their lengths variable, so that each one in order can be set so as to be equal to the coefficient of the corresponding term; moreover, by mechanism each one makes with the last the same angle that the first makes with the axis of x. Thus, on rotating the first, the end of the last describes a curved line cutting at one or more points the axis of y; every angle θ of the first which causes this intersection is noted, and the value being inserted in the original equation gives at once one of the real roots.

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* Read December 12, 1885.

† 'Messenger of Mathematics,' 1873, vol. ii. p. 51.

I am not aware of any other machines for solving equations of a high order, except Mr. Cunynghame's, which will be described this afternoon. Machines for solving simultaneous equations also exist.

Like Mr. Hinton, I made use of the equality of moments by arranging a series of levers to operate upon one another. Thus, let the levers be called successively 1, 2, 3, 4, &c.; then 1 is on a stationary axis; it has at unit distance from its axis and on either side of it pivots, from each of which hangs a pan or hook labelled $-a$ and $+a$. If, then, a weight of a units is put upon either of these pans, the moment of the force upon the beam will be $-$or $+a$ units.

Let a second beam 2 be connected with 1 by a sliding joint which is permanently at unit distance from the axis of 2; let the joint also carry a scale-pan, and let there be another scale-pan at unit distance on the other side. These are labelled $+b$ and $-b$. A weight of b units hung on either of these pans will produce a turning moment on b of $+$or$-b$ units, and a turning moment on a of $+$ or $-bx$ units, where $x+1$ units is the distance between the axes of 1 and 2. Such a pair of beams of course will solve a simple equation whether the root be positive, negative, whole, or fractional; for as the second beam is made to traverse, it must pass some position where $a \pm bx=0$, and on the two sides of this position the moments on 1 will be opposite in sign. Therefore by shifting the beam till the arm changes position, the root can be exhibited by a pointer on a scale.

Similarly, if a third beam be mounted opposite 1 with a fixed axis, and if it be provided with pans as before, labelled $-c$ and $+c$, and if this be connected with 2, as 2 is with 1, then on pushing 2 along between these two beams a weight of c units on one of the c pans will exert on 2 a moment of $\mp cx$, and on 1 of $\mp cx^2$ units. If, then, a quadratic equation of the form $a+bx+cx^2=0$ has to be solved, weights of a, b, and c units are placed on the proper pans and the beam 2 slid along until the beam 1 shifts its position. The two places at which this occurs are the two real roots of the quadratic. When the quadratic has no real roots, the machine can be still employed to find the impossible roots, as will be explained later.

A fourth beam placed opposite to 2 and connected with 3, as 2 is with 1, will in the same way make the machine capable of finding the real roots of a cubic equation, and so by adding more beams to the stationary and movable sets, an equation of any degree can in theory be solved. Of course, as the number of joints increases, the friction and elasticity of the working parts increase enormously, and so with the best work possible there is a practical limit beyond which it is hardly possible to go.

The only difficulty lay in the design of the beams, which would allow the connections between them to be capable of continuous variations at all parts of their length. The joint must be able to slide freely past the axis and past the coefficient pans. All the beams are of the same construction except that alternate ones have the connecting joint on opposite sides of the centre, as shown by the diagram fig. 1, Pl. V. Fig 2 is a front view of a beam with its supporting-rod and coefficient hooks complete; fig. 3 a back view, and fig. 4 a plan of the same. Fig. 5 is a transverse section through the middle, and fig. 6 an end view of a beam and connecting joint but without supporting bar. It will be seen that there runs along the back of each beam a web which may be embraced at any part by a pair of fingers fixed to the front of the next beam behind it, *i. e.* by the beam which is next higher in order of number.

It is evident that this construction allows the connecting joint of each beam to slide past the centre of the next lower one.

The supporting bars hang from a series of longitudinal beams lying in a cradle, the alternate ones being joined together. These then can be shifted relatively past one another, and one set can carry an index reading on a scale on the other set, so placed as to read zero when the distance of the centres of consecutive beams is one unit.

To show how to apply the machine to find the impossible roots of a quadratic, it will be necessary to consider the quadratic equation $y=a+bx+cx^2$ as representing a parabola. In this parabola a has nothing to do with the shape of the curve. Variation in a only shifts it or makes it slide up and down the axis of y. It may therefore be struck out for the present. Then the parabola passes of necessity through the intersection of the axis of x and y. If the sign of b is changed, the new parabola will be a reflection of the old about the axis of y. Fig. 7 thus represents the double equation $x^2 \pm bx = y$.

The distances of the vertices of these parabolas from the axis of y are $\mp \frac{b}{2}$, and the values of y at the vertices are each $-\frac{b^2}{4}$. Now let a line parallel to the axis of x slide down the axis of y, and let its distance above the axis of x be a in a series of equations $a + bx + (c)x^2 = y$ (the c may be omitted), then those values of x at which this line cuts the parabola will be roots of the equations. As it slides down, that is as a increases algebraically, it approaches the vertex, and the two roots become more nearly equal until at the vertex they join, and there are two equal roots; here of course $b^2 = 4a$. If the sliding line is below this position, it misses the parabola, and there are no real roots.

Now going back to the case of real roots, it is clear that they are equally greater and less than $\mp \frac{b}{2}$, which is the distance of the vertex of the parabola from the axis of y, and the excess or defect is the half width of the parabola at the place of cutting. Now this half width is by the nature of the parabola equal to the square root of the distance along the axis from the vertex to the sliding line. If, then, the sliding line misses the parabola, the half width of the curve at the position of the line being equal to the square root of its distance above the vertex, will, as this distance is now negative, be impossible, or the curve at this region does not exist; nevertheless the imaginary half width is still the square root of the distance of the sliding line *above* the vertex. Geometry is incapable of representing this impossibility; but it is curious that the machine can be made to indicate its existence and measure its amount. Since the ordinates of the parabola represent the moments on the first beam for every value of x, it is clear that a minimum of moment will be produced in the case of impossible roots. If, then, the machine fails to find two real roots (which of course must be within its limits), the first beam must be constrained by a spring instead of being free. The deflection will then indicate moments, and therefore the position of the minimum may be observed. The reading on the scale of the instrument is then the real part of the root, and the square root of the moment, which may be found by applying weights, the impossible part.

In the case of a cubic equation there must be one real root, and there may be three; if only one exists this may be found and divided out, when the resulting impossible quadratic can be solved by the machine. If all these roots are real, they can of course be found directly.

In a biquadratic, or one of a higher order, the possible roots may be found. If there remain more than two impossible roots, the machine is incapable of finding them.

While attempting to find properties of such equations that might be made use of in the construction of a machine, I found a curious relationship between two pairs of curves which might be turned to account in a curve-tracing machine. They are probably well known, but if so it may be worth while to mention them, as they have an interest in connection with this paper.

If the signs of all the terms of an equation $bx+cx^2+dx^3$ $+$ &c. $=y$ be changed, the curve of course is in no way changed. If the signs of the even powers be changed, the curve is obviously turned round about the origin through two right angles; but if the sign of the term containing x alone be reversed, the shape of the curve is entirely altered, yet the relation between the new and the old curve is very simple.

Firstly, they cross at the origin, making there equal angles on either side of the axis of y. Secondly, whatever value of x be taken the tangents of the two curves for that value will cross one another and the axis of y in the same point. This must be so, for the distance of this point from the origin $=y-x\frac{dy}{dx}$, and in this expression no part of the term involving x remains. Fig. 8 shows the series of curves for the particular series of equations

$$y=x^3+3x^2+2x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$y=x^3-3x^2+2x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$y=x^3+3x^2-2x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$y=x^3-3x^2-2x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and shows the intersection of three pairs of tangents in y.

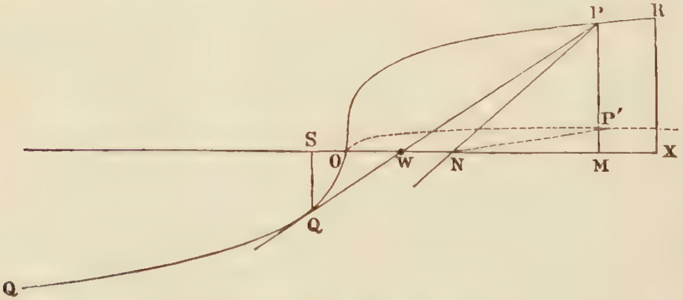
I should say, in conclusion, that I have brought this machine before the notice of the Physical Society not as a thing which is likely to be of any practical use, but because it has a certain interest, more especially in connection with another machine invented by my friend Mr. Cunynghame, which is really practically valuable, and which he will show this afternoon. (See below.)

XXXVIII. *On a Mechanical Method of Solving Quadratic and Cubic Equations, whether the Roots be real or impossible.*

By HENRY CUNYNGHAME, *Barrister-at-Law*[*].

THE method which I have the honour to bring before this Meeting depends upon the use of a parabola of the form represented by the equation $x^n=y$. And it is capable of effecting solutions of all equations of the form $x^n+mx=C$.

Let us first examine the case of a cubic equation. By Cardan's rule reduce the equation to the form $x^3+Ax+B=0$. In fig. 1 let P O Q be a cubical parabola, such that the ordinate at any point represents the cube root of the abscissa along the axis of X, measured negatively towards the right. That is to say, let $OM=(PM)^3$. From P draw any line such



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[*] Read December 12, 1885.

that the angle $\mathrm{PNM} = \cot^{-1}\mathrm{A}$, and let $\mathrm{ON} = \mathrm{B}$. Then it is clear that

$$\mathrm{OM} + \mathrm{MN} + \mathrm{ON} = 0\,;$$

that is,

$$(\mathrm{PM})^3 + \mathrm{PM}\cot\mathrm{PNM} + \mathrm{ON} = 0\,;$$

or, putting $\mathrm{PM} = x$, $\cot\mathrm{PNM} = \mathrm{A}$ and $\mathrm{ON} = \mathrm{B}$, we have

$$x^3 + \mathrm{A}x + \mathrm{B} = 0.$$

And this, therefore, shows that if from a point N, such that $\mathrm{ON} = \mathrm{B}$, we draw a line NP inclined at such an angle with OX that $\tan\mathrm{PNM} = \mathrm{A}$, then the value PM will represent a root of the equation $x^3 + \mathrm{A}x + \mathrm{B} = 0$. Of course, if PN produced cuts the other branch of the cubical parabola, there will be two other real roots. If it does not, those other roots will be impossible.

In making a machine practically, it is well for convenience to draw on a paper a cubical parabola, say 2 feet wide, and make 1000 divisions on OX each way. Then make $\mathrm{XR} = \sqrt[3]{1000} = 10$. But as this would make the parabola too thin, let us multiply its ordinates by 10, and then take a cotangent protractor scale with the cotangents also multiplied by 10. So that the parabola drawn, as well as the tangent scale in any position, is an orthographic projection of the real one. Such a machine will find real roots true to two places of decimals.

To find the impossible roots. First find the real root, say P; draw P Q to touch the other branch of the parabola in Q. It may then be easily shown (though I am not aware that it has been before pointed out) that

$$\mathrm{SQ} = \frac{\mathrm{PM}}{2}\,;$$

and that therefore

$$\mathrm{OS} = \mathrm{SQ}^3 = \frac{\mathrm{PM}^3}{8}.$$

And, further,

$$\frac{\mathrm{SW}}{\mathrm{MW}} = \frac{\mathrm{SQ}}{\mathrm{PM}} = \frac{1}{2},$$

and

$$\mathrm{SW} + \mathrm{MW} = \mathrm{SO} + \mathrm{OM} = \frac{\mathrm{PM}^3}{8} + \mathrm{PM}^3 = \tfrac{9}{8}\,\mathrm{PM}^3,$$

whence

$$3\text{SW} = \tfrac{9}{8}\,\text{PM}^3\,;$$

and

$$\text{SW} = \tfrac{3}{8}\,\text{PM}^3,$$

and

$$\text{MW} = \tfrac{6}{8}\,\text{PM}^3.$$

Now if α be the real root of the equation, so that $\text{PM} = \alpha$; then, by dividing $(x - \alpha)$ into $x^3 + \text{A}x + \text{B} = 0$, we can easily show that the other two imaginary (or real) roots are given by the expressions

$$-\frac{\alpha}{2} \pm \sqrt{-\tfrac{3}{4}\alpha^2 + \text{A}},$$

B being $= \alpha^3 + \text{A}\alpha$. But we have already shown that

$$\tfrac{3}{4}\alpha^3 = \tfrac{6}{8}\,\text{PM}^3 = \text{MW},$$

and we know that $\text{A} = \dfrac{\text{MN}}{\text{MP}}$; wherefore

$$\sqrt{\text{A} - \tfrac{3}{4}\alpha^2} = \sqrt{\frac{\text{MN}}{\text{MP}} - \tfrac{3}{4}\,\frac{\text{MW}}{\text{MP}}} = \sqrt{\frac{\text{WN}}{\text{MP}}}\,;$$

which gives us this curious result—that if PM be the real root of the equation, the other two roots are

$$-\frac{\text{PM}}{2} \pm \sqrt{\text{ratio of WM : PM}}.$$

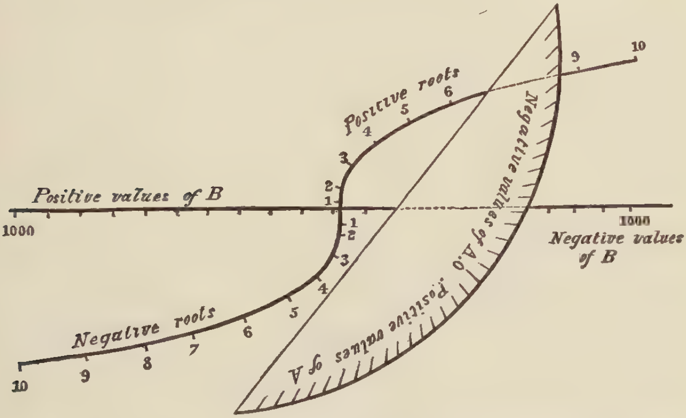
If W lies to the left of N, then WN is negative and the roots are impossible. If it lies to the right of N then the roots are real; and of course the ratio WN to PM is easy to find with the tangent scale; for it is the difference of the cotangents of the angles PNM and PWN, and may be read off at once by sliding the cotangent scale so as to have its centre at W and its edge parallel to NP; or, in fact, in any convenient manner.

It remains only to add, that a geometrical relation can always be found for the impossible parts of the roots of all equations given by the form $x^n + mx = \text{C}$. Thus, in a parabola the impossible part would be represented by the horizontal distance of the line PN, measured along the axis of X from the tangent to the curve drawn *parallel* to PN, and would thus be the square root of a line, not of a ratio.

The same instrument may be used to take square roots of any numbers, so as to find the square root of the ratio above mentioned, by holding the cotangent protractor so that its centre is at O, and the division on its scale corresponding to the square of which the root is required is on the axis of X. Then we shall have $\frac{\text{OM}}{\text{PM}} = \cot\alpha =$ division on the scale. But $\text{OM} = \text{PM}^3$; $\therefore$ division on the scale shows $\frac{\text{PM}^3}{\text{PM}}$, that is to say PM^2, whence PM gives the square root required. The values of PM should be written off in divisions and decimals along the branches OP and OQ, one being negative, the other positive; and the sides of the protractor, and also the axes OX, OX′, should be marked with their proper signs.

It will be observed throughout that I have treated lines and areas and ratios of lines as representing numerical values. Hence the equations above given have numerical values for x; and to make all the terms of the equations of the same order, constants would have to be introduced, which I have left out because they are unnecessary.

The following is a sketch of the machine represented in the act of solving the equation $x^3 - 20x - 200 = 0$.



(One of these instruments is to be seen at the loan collection, South Kensington, where it will remain permanently.)

XXXIX. *Note on some Organic Substances of High Refractive Power.* *By* H. G. MADAN, *M.A.*[*]

IN the course of some correspondence respecting M. Bertrand's polarizing-prisms, one of which I had the honour of showing to the Society last summer, I was informed that the cement used was naphthyl-phenyl-ketone dibromide. It was evident, from the theory of the prism, that this substance must have a refractive index at least as high as that of calcspar for the ordinary ray, viz. 1·658; and as so highly refractive a cement seemed likely to be of great value for many purposes, I thought it worth while to prepare some of the ketone and examine its properties. I now exhibit a specimen of it.

I prepared it by the general method devised by Grucarevic and Merz (*Berichte der deutschen chemischen Gesellschaft*, vi. pp. 60 & 1238) for obtaining double ketones; viz. by heating together benzoyl chloride and naphthalene, and placing in the mixture a strip of zinc. The product was fractionally distilled, to free it from the excess of naphthalene, and was finally obtained as a thick yellow oil, boiling at a point so nearly that of mercury that a mercury-thermometer could not be used to determine it. In properties it seems a very stable, neutral, harmless substance like Canada balsam; but unfortunately it does not appear capable of hardening like balsam, and hence it is not by itself adapted for a cement. It is insoluble in water, but dissolves readily in alcohol and benzol.

Its refractive index was determined in the usual way, a hollow prism (of refracting angle of 59° 48′) being filled with it, and the angle of minimum deviation for yellow sodium-light observed with a refractometer; from which data the usual formula gave as its refractive index, 1·666. This is even higher than that of carbon disulphide (1·63), and very nearly the same as that of calcspar given above.

I compared its dispersive power with that of a prism of very dense glass (refractive index 1·73) having a refracting angle of 60°, with the following results:—

---

[*] Read January 23, 1886.

Fraunhofer's line.	Ketone. $\mu=$	Glass. $\mu=$	Carbon disulphide. $\mu=$
B . . .	1·654	1·725	1·617
E . . .	1·678	1·744	1·643

From this it is easily seen that the coefficient of dispersion of the ketone is 1¼ times that of the glass, and almost exactly the same as that of carbon disulphide.

I have made the compound of the ketone with bromine, referred to by M. Bertrand, but I hesitate to recommend it as a cement, at any rate for anything made of calcspar; as it seems liable (like many similar bromides) to decompose with formation of hydrobromic acid, which of course acts upon the spar.

The ketone itself has a refractive index quite sufficiently high, if only some means can be found of hardening it.

I cannot, however, yet vouch for its permanency. It was made in August last, and remained for several months without change; but it has lately shown a tendency to pass from the colloidal to a crystalline condition, especially during the cold weather, although I had previously exposed it to a temperature of —20° without causing any alteration beyond an increase of viscosity.

I would also ask permission to show the Society a specimen of another highly refractive substance, viz., metacinnamene. It is a polymeric form of cinnamene, obtained by the action of light or heat upon the latter substance.

Cinnamene is a colourless liquid, obtained by distillation from the resin storax. Its refractive index I found to be 1·54 for the D line, nearly the same as that of Canada balsam. When this liquid is exposed to light for a few weeks, or heated to 190° in a sealed tube for half an hour, it becomes a glass-like solid, contracting greatly during the change.

In order to determine its refractive index, it was heated until it became viscous and placed in a hollow prism. The angle of minimum deviation for yellow sodium-light was found to be 39° 44′ 35″; whence its refractive index is calculated to be 1·593. This is much higher than that of any other resin, so far as I can make out; and metacinnamene would make a very valuable cement, since it is readily softened by heat and becomes remarkably hard and tough on cooling, but it does not show any great adhesiveness for glass. I hope to get good results by mixing it with other things, such as the ketone above described.

Another highly refractive organic substance is monobromonaphthalene, of which I exhibit a specimen.

It is a nearly colourless liquid, more stable than similar compounds containing bromine usually are, and boiling at the high temperature of 285° C. Its refractive index for the D line is 1·662; very nearly equal to that of naphthyl-phenyl-ketone. I am rather surprised that it has not come into use as a substitute for carbon disulphide for filling prisms, as it is much less volatile and inflammable than the latter substance, while it has an even higher refractive and dispersive power.

The great desideratum at present is, a substance which has all the excellent qualities of Canada balsam—colourless, neutral, permanent in the air, becoming fluid when moderately heated, but hard and tough when cold, and with a refractive index of *at least* 1·66.

Such a substance would not only be of great use in the construction of polarizing and other prisms, but it would also be invaluable as a medium for mounting microscopic objects; since details of structure are brought out much more clearly when the object is immersed in a medium which differs greatly from it in refractivity. Phosphorus, arsenic sulphide, and mercury-potassium iodide have been used for this purpose, but they are all open to grave objections. None of them are permanent in the air; some are dangerously inflammable; while most of them act on delicate organic structures, although available for such things as siliceous Diatomaceæ.

The most hopeful direction in which to look is undoubtedly towards some of those complex organic compounds which are now being built up by many workers in England and Germany.

XL. *Note on the Determination of the Volume of Mercury in a Thermometer.* *By* A. W. CLAYDEN, *M.A., F.G.S., Bath College*[*].

IN Professor Clark's paper on the Heat-capacity of a Thermometer[†], he shows that it may be calculated approximately if the specific gravity of the instrument, and that of the glass of which it is made, be known. The first step in his method is to determine the volume of the contained mercury, which he points out may be ascertained by the following formula,

$$V_1 = \frac{V(S-S_2)}{S_1-S_2}; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which V_1 = the volume of mercury,
S_1 = its specific gravity,
V = the volume of the instrument,
S = its specific gravity,
S_2 = the specific gravity of the glass.

Now this expression will only be correct if the bulb and tube of the thermometer be completely filled with mercury, as they will be at a temperature (t°) slightly above the highest reading of the scale. At all lower temperatures V will be too large and S too small; an error being introduced, as Professor Clark points out, by the empty space of unknown capacity which occupies the unfilled portion of the tube.

At this higher temperature (t), V_1 becomes $V_1(1+\beta t)$, β being the absolute coefficient of expansion of mercury. Similarly V becomes $V(1+\alpha t)$, α being the coefficient of cubical expansion of the glass.

Also

$$S \text{ becomes } \frac{S}{1+\alpha t}; \quad S_1 \text{ becomes } \frac{S_1}{1+\beta t}; \text{ and } S_2 \text{ becomes } \frac{S}{1+\alpha t}.$$

If, now, these results be substituted for those in formula (1), the expression obtained will be theoretically correct, as the empty space vanishes at t°.

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[*] Read January 23, 1886.

[†] Proc. Physical Soc. vol. vii. p. 113. [Phil. Mag. vol. xx. p. 48.]

Thus

$$V_1(1+\beta t)=\frac{V(1+\alpha t)\left(\dfrac{S}{1+\alpha t}-\dfrac{S_2}{1+\alpha t}\right)}{\dfrac{S_1}{1+\beta t}-\dfrac{S_2}{1+\alpha t}},$$

which, by simplification, becomes

$$V_1=\frac{V(S-S_2)}{S_1-\dfrac{S_2(1+\beta t)}{1+\alpha t}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It will be seen that (2) gives rather a larger result than (1), because β is greater than α. This, however, is what must be expected if reference is made to (1). There V is too large and S is too small, so that the product VS remains unaltered; because $S=\frac{W}{V}$ if V is the volume in cubic centimetres, whence $VS=W$; but the product VS_2 becomes greater as V increases. Hence (1) gives too *small* a value for V_1, a result which is compensated for in (2).

XLI. *Note on the Paper on some Thermodynamical Relations by* Prof. W. Ramsay *and* Dr. S. Young. *By* Professors W. E. Ayrton, *F.R.S., and* John Perry, *F.R.S.*[*]

We have written this note partly for the purpose of drawing attention to the fact that, although scientific work of many kinds is being much more thoroughly done now than ever before in the history of the world, mainly because the workers are specialists, yet a great deal of valuable work is rendered comparatively useless because the specialist is now too perfect a specialist, and so obtains no help from other departments of physics than that in which he himself is working.

We have ourselves, for example, wasted much time in useless observation of what seemed to be capricious behaviour of voltaic cells, a result to be expected when electricians who are not practical chemists conduct chemical investigations. Again, the specialist in pure mathematics might often do

[*] Read January 23, 1886.

much more valuable work if he had practised the application of his mathematics to natural phenomena, for he would learn of new directions in which to conduct his special investigations. We have been told by certain authorities in Chemistry that a few chemists with a good working knowledge of mathematics possess at the present time facilities for rapidly improving our knowledge of chemical science.

Drs. Ramsay and Young must forgive us for making their recent papers the text of a discourse of this kind. We might have selected papers by other authors which show more clearly the truth of what we say; but the work done by these two gentlemen is of such a valuable kind, their scientific positions are so well established, that we can venture to take this liberty without feeling that we do any harm in making an illustration.

These gentlemen read a paper at the last meeting of the Physical Society, in which they gave the gist of two papers, one of which had been published in the December number of the Philosophical Magazine, and the other has since been published in the January number. Members of the Society will find the first of these in the number of the 'Proceedings' of our Society which reached them yesterday. Referring to the papers in the Philosophical Magazine and to the published abstract of the paper read before the Physical Society, we have for saturated vapours :—

I. Ramsay's first law :

$\frac{L}{s_1 - s_2}$ is constant for all substances at the same pressure; L being latent heat, and s_2 and s_1 the specific volumes of the liquid and saturated vapour.

II. Ramsay's second law :

$\frac{L}{s_1 - s_2}$ at pressure p_1 bears a constant ratio to $\frac{L}{s_1 - s_2}$ at pressure p_2 for all substances.

III. Young's law :

$t\frac{dp}{dt}$ is constant for all substances at any given pressure; t being the absolute temperature, and p the pressure of a saturated vapour.

Their paper, published in the December number of the Philosophical Magazine and in our 'Proceedings,' is devoted to the illustration of these three laws, and it is the record of an extraordinarily large amount of calculation.

IV. Ramsay and Young's law:

$\frac{d}{dp}\left(t\frac{dp}{dt}\right)$ is nearly constant for all stable substances at the same pressure.

One of us pointed out in the discussion on the paper, that to measure $\frac{dp}{dt}$ by drawing a tangent to a plotted curve was usually an inexact method; and that much greater accuracy would have been obtained by the authors if they had, for each substance, found the values of the constants α, β, and γ in Rankine's well-known formula

$$\log p=\alpha-\frac{\beta}{t}-\frac{\gamma}{t^2}+\&c.; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and that probably the best way of making a comparison between two substances and observing changes due to dissociation would be by remarking that, for the substances examined by Regnault,

$$\alpha-\frac{\beta}{t}-\frac{\gamma}{t^2}+\&c.=a-\frac{b}{\theta}-\frac{c}{\theta^2}+\&c., \quad . \quad . \quad . \quad (2)$$

if t and θ are the absolute temperatures of two saturated vapours at the same pressure. It is unlikely that any person will discover an improvement on Rankine's formula just given. It cannot merely be looked upon as an empirical formula being based on Rankine's molecular theory. The exactness with which Rankine has reproduced Regnault's numbers, using only three constants, is exceedingly remarkable. Rankine in his paper refers to the formula

$$\log p=\alpha-\frac{\beta}{t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which is the same formula but with two constants only, and is inexact for calculations of pressure.

To show that Messrs. Ramsay and Young might have employed a less laborious method of testing, we may first point out that it would have reduced the labour by something like 75 per cent. to have recognized the fact that I., II., III., and IV. are all identical. If any one of them is true, they must all be true. Thus III., when stated mathematically, is simply

$$t\frac{dp}{dt}=\phi(p), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$\phi(p)$ being the same function of the pressure for all substances. Differentiating (4) with regard to p, we obtain at once law IV. If it had sooner been observed that

$$\frac{L}{s_1-s_2}=\frac{t}{J}\frac{dp}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

is simply a statement of the Second law of Thermodynamics, the authors would probably have examined the truth of the law in the shape III. only; since the specific volumes of vapours are so difficult to test experimentally, that Rankine, to obtain the density of steam, calculated s_1-s_2 by dividing the latent heat L by $\frac{t}{J}\frac{dp}{dt}$ rather than depend on direct measurement. Again, since the first law is $\frac{L}{s_1-s_2}=\phi(p)$, if we insert L_1 and L_2, p_1 and p_2, s and σ, we see that law II. is identical with I., and therefore I., II., III., and IV. are merely different methods of stating the same law, and to test one is to test them all.

Now we are sorry to say that we should have not attempted to test any of these forms of the law. The form which may be tested with most accuracy is III., or, as it may be stated mathematically, as in (4)

$$t\frac{dp}{dt}=\phi(p),$$

or

$$\frac{dp}{\phi(p)}=\frac{dt}{t},$$

or

$$t=a\psi(p), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where t is the absolute temperature corresponding to the pressure p of a saturated vapour; a is a number which depends on the nature of the substance, and the function $\psi(p)$ is the same for all substances. Hence to test laws I., II., III., and IV. it is simply necessary to see whether (6) is true, as (6) is identical with them. Now if (6) is true, it follows that the ratio of the temperatures of two vapours to one another at any pressure is the same as at any other pressure, or

$$\theta = kt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In fact, then, we can test laws I., II., III., and IV. by simply testing (7); and, as we find from Regnault's or Rankine's formula (1) or (2) that (7) is untrue, there is nothing further to be said about the four laws in question.

We draw attention to the fact that Dalton's law, which was the earliest law concerning saturated vapours, and of whose inaccuracy the text-books always warn us, is a much more correct law than the four identical laws which we have been considering. For, to state Dalton's law mathematically,

$$\frac{dp}{dt} = \phi(p). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

That is, the rate of change of pressure with temperature is constant at a given pressure for all saturated vapours. Integrating (8), we obtain Dalton's law in the shapes

$$t + a = \psi(p), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\theta = t + b. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

That is, there is a constant difference between the temperatures of two saturated vapours at the same pressure. Now this law has been known to be incorrect for a long time; but if, assuming this law to be correct, we calculate the temperatures of saturation (corresponding to various pressures) of, say, bisulphide of carbon from those of water, we shall find our errors to be much smaller than those obtained when we use any of the above-mentioned four laws.

Of the law connecting θ and t, which has been published in the January number of the Philosophical Magazine, it is easy to see that the two forms in which it has been stated by the authors are inconsistent with one another. Such inconsistency is not of much importance in the statement of empirical laws; but we think that one of the forms is preferable to the other on account of its symmetry, as it may be put in the form:—The reciprocals of the absolute temperatures of two saturated vapours at corresponding pressures are linear functions of one another, or

$$\frac{m}{t} + \frac{n}{\theta} = 1.$$

Or, again, it may be put into the shape

$$\phi(p)=\alpha-\frac{\beta}{t}=a-\frac{b}{\theta},$$

which may be compared with (3).

In conclusion, it will be observed that in calculating the temperature of a saturated vapour, a small-looking error may represent an error which looks much larger in the pressure. In comparing the errors of calculation from other empirical formulæ with those due to Rankine's formula, it is well to remember that Rankine calculated pressure, and that his errors, which for that reason might have been expected to appear large, were very small, thus showing the great accuracy of his formula.

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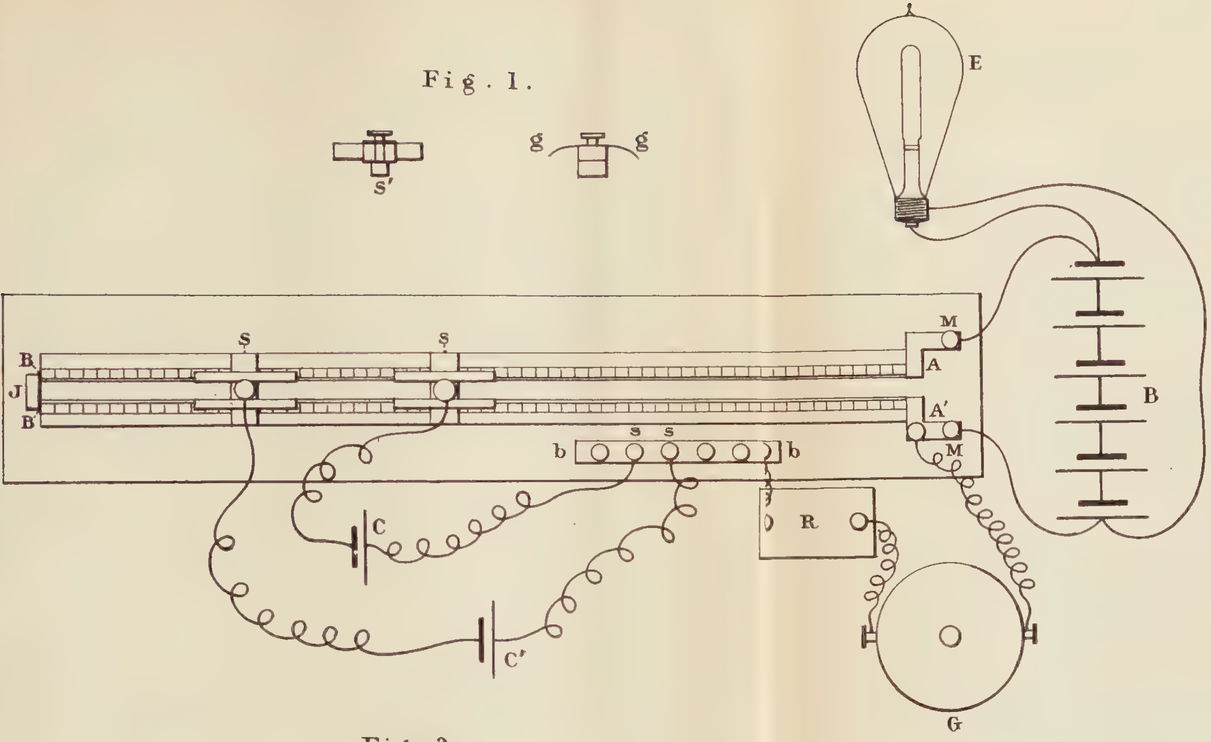


Fig. 1.

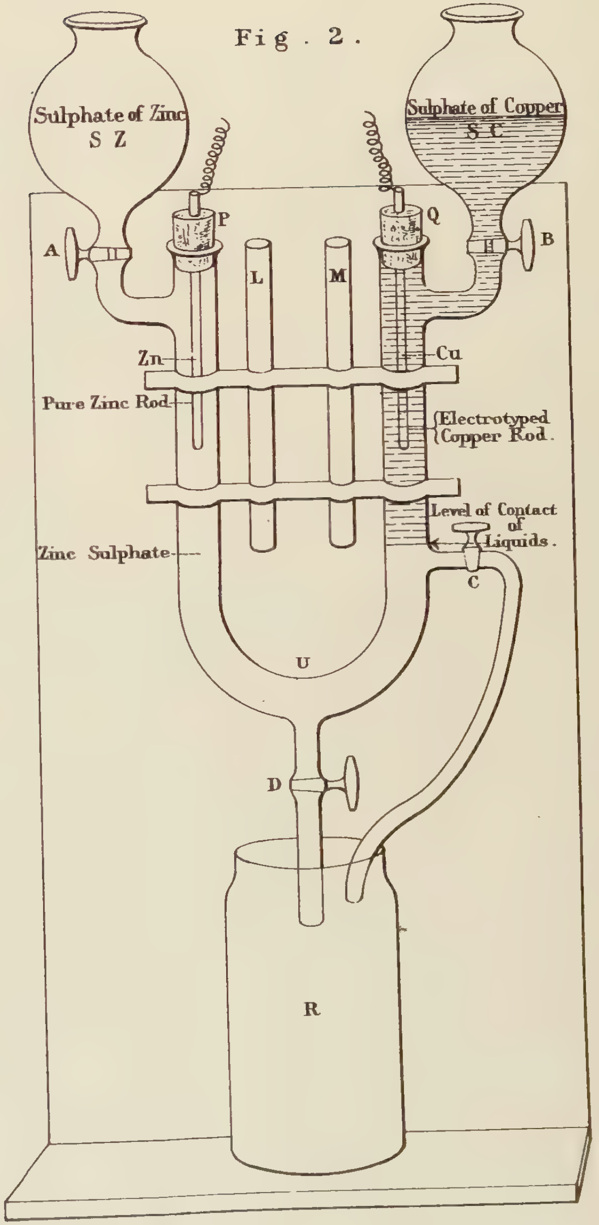
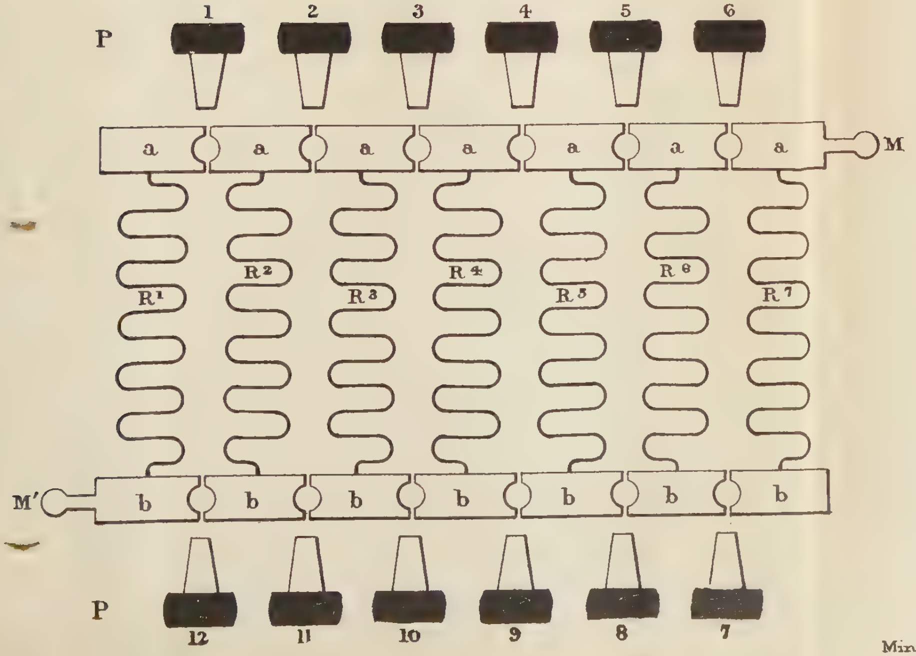


Fig. 2.



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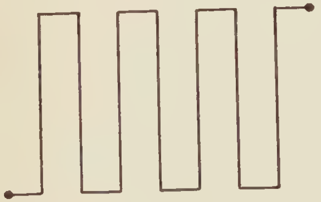


Fig. 4.

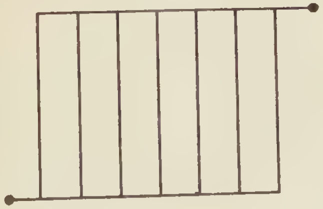
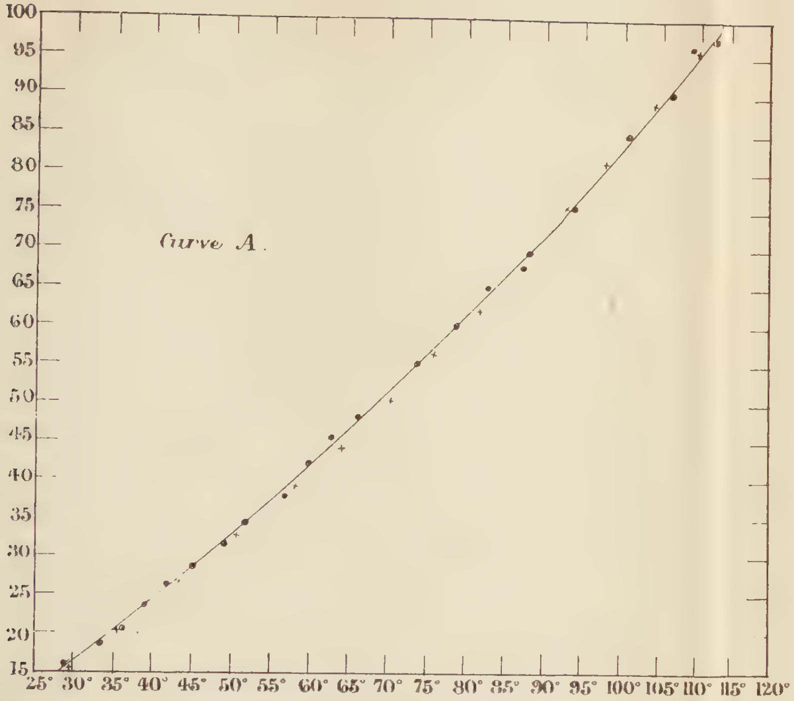
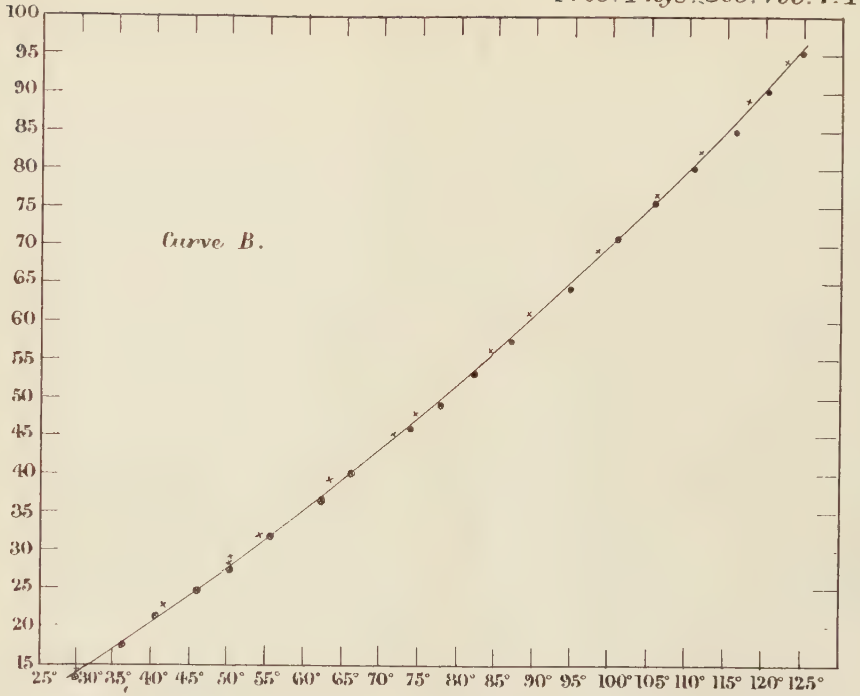
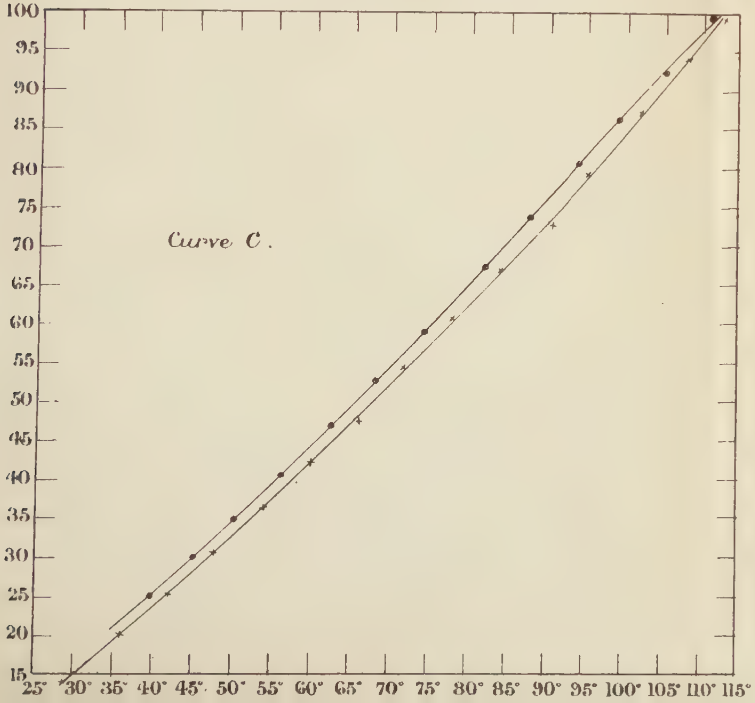


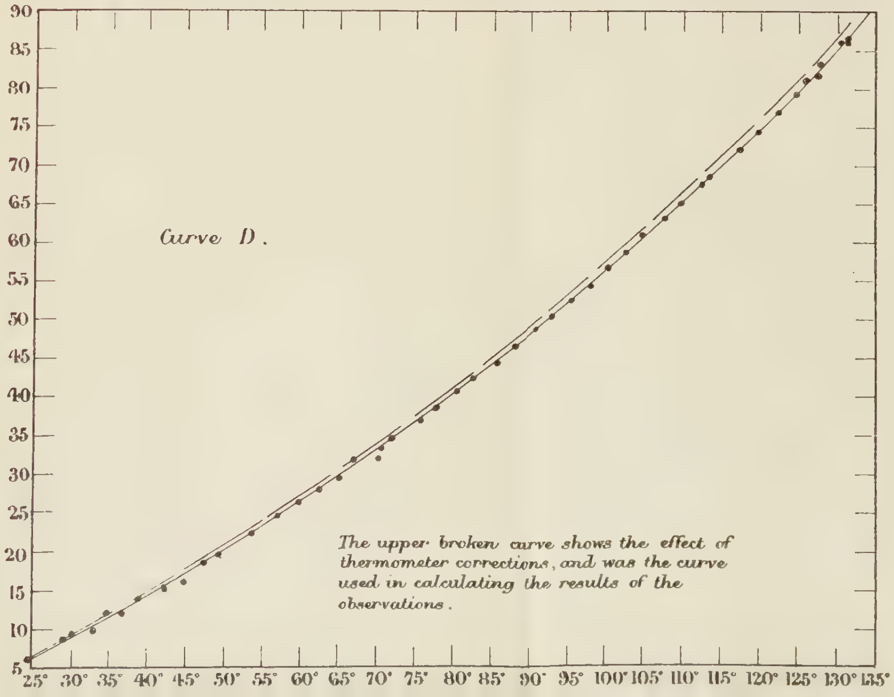
Fig. 5.

Mintern Bros. lith.









The upper broken curve shows the effect of thermometer corrections, and was the curve used in calculating the results of the observations.

Fig. 1.

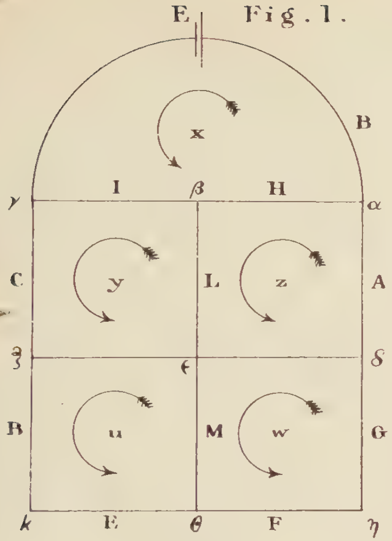


Fig. 2.

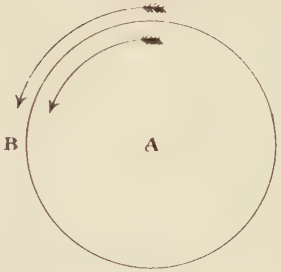


Fig. 3.

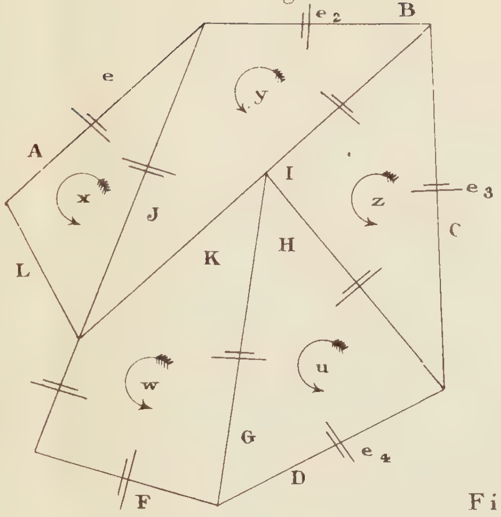


Fig. 4.

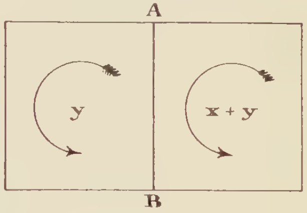


Fig. 5.

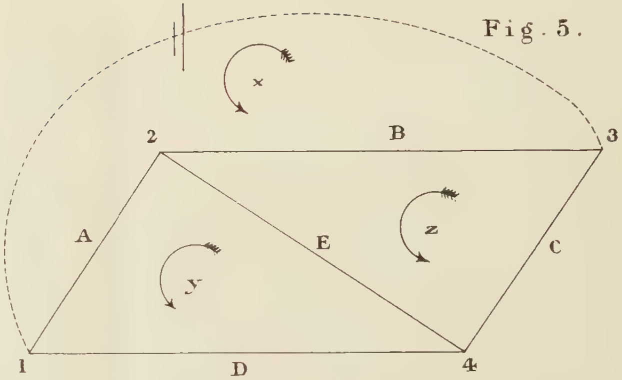


Fig. 6.

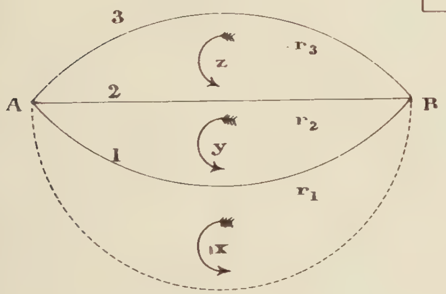


Fig. 7.

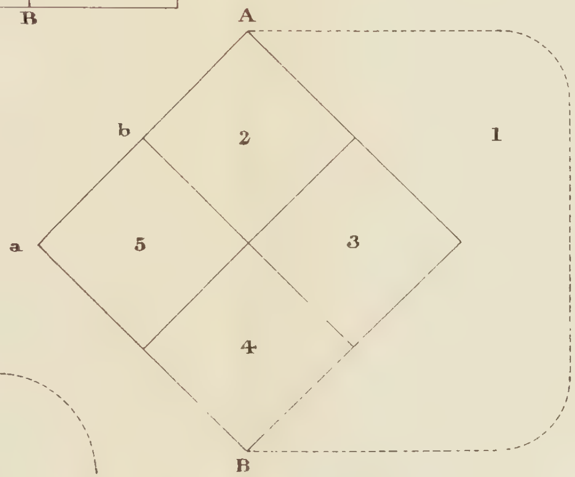


Fig. 8.

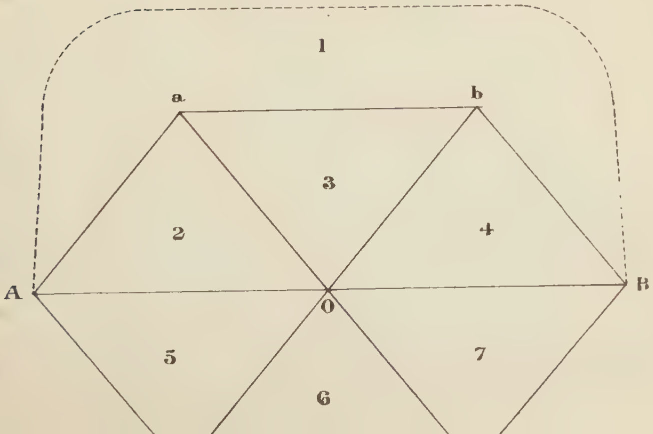


Fig. 9.

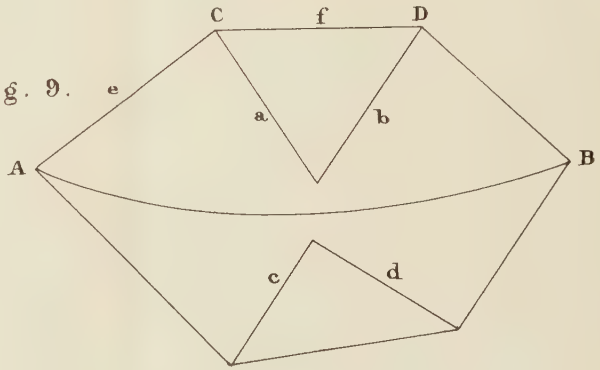


Fig. 10.

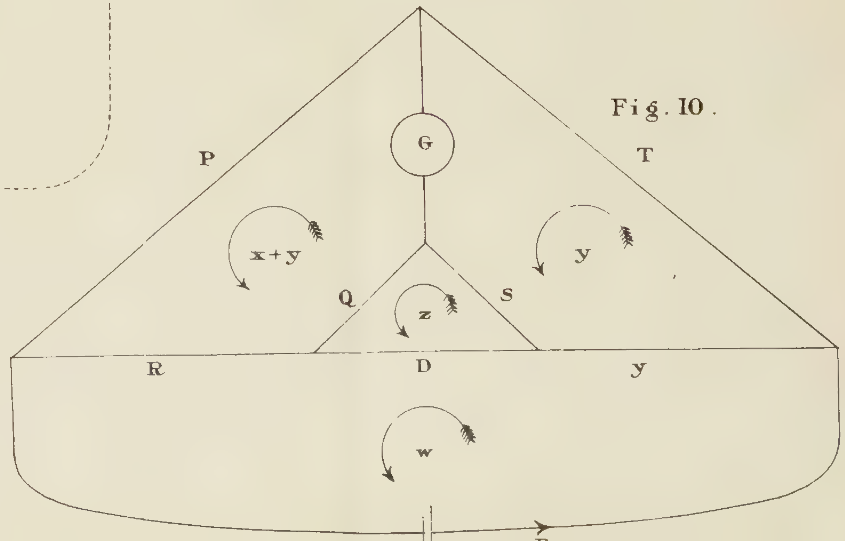


Fig. 11.

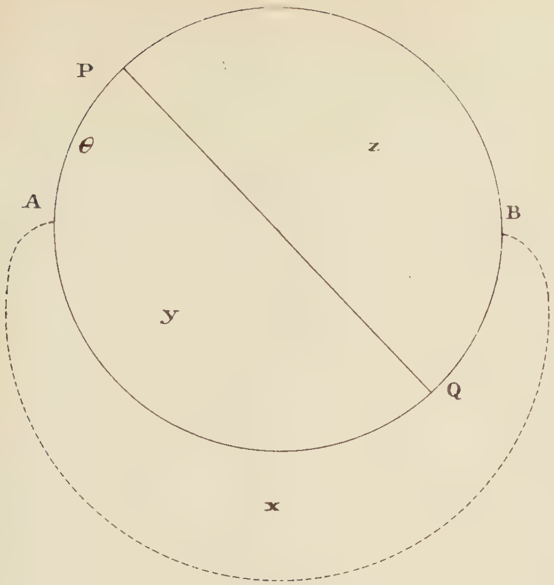


Fig. 16.

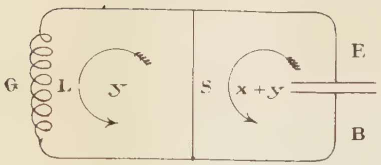
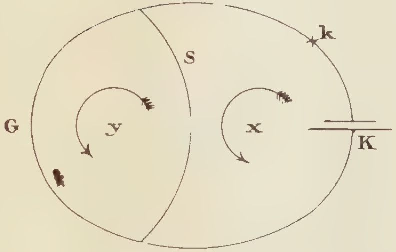


Fig. 17.



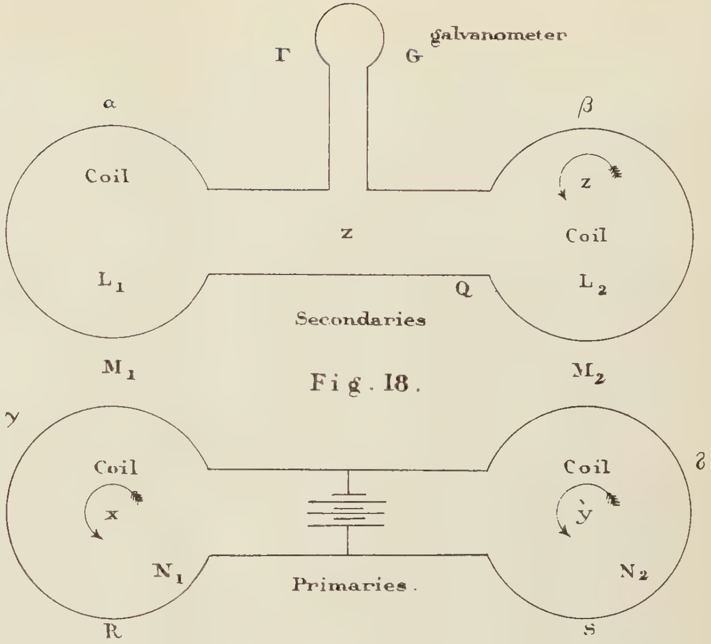


Fig. 18.

Fig 12

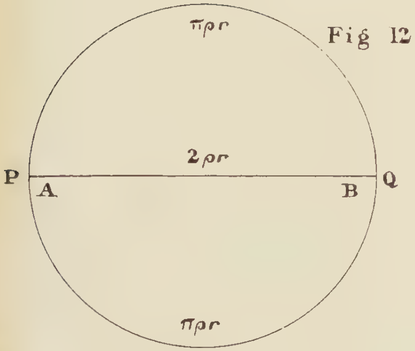


Fig. 13.

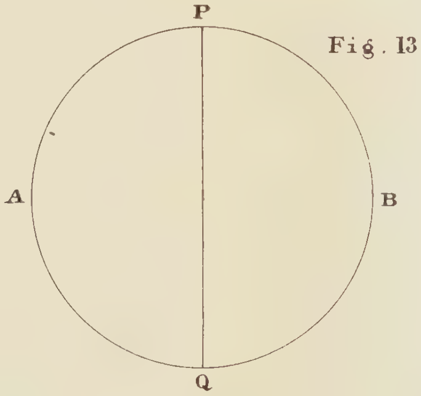
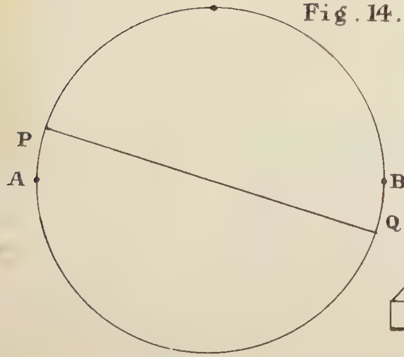


Fig. 14.



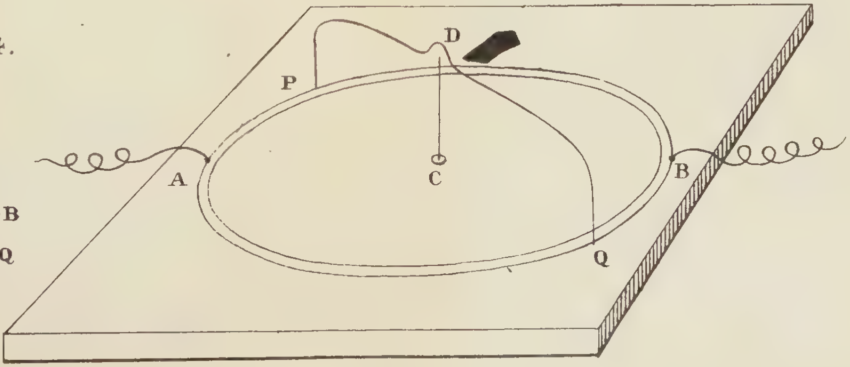
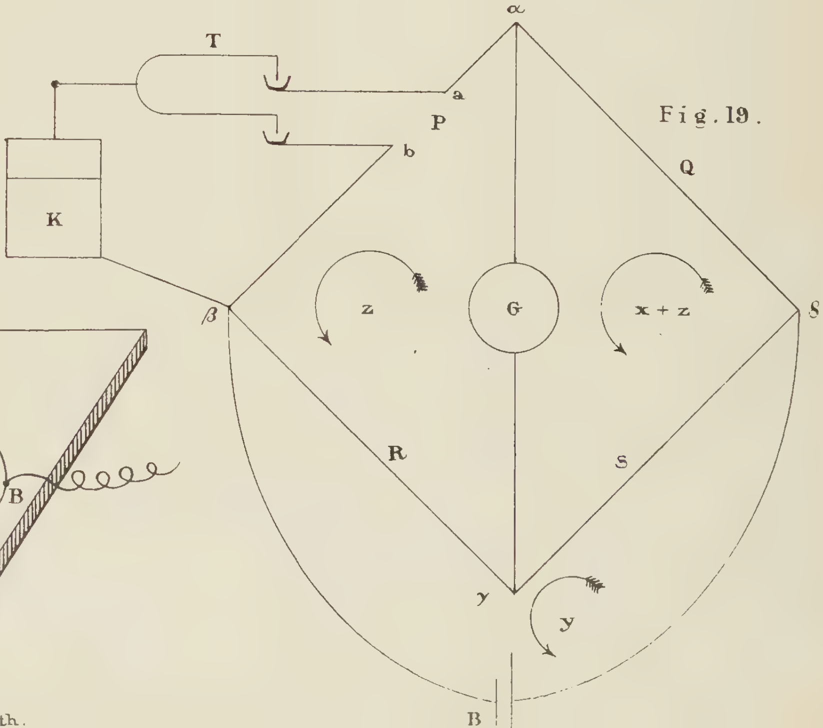


Fig. 15.

Fig. 19.



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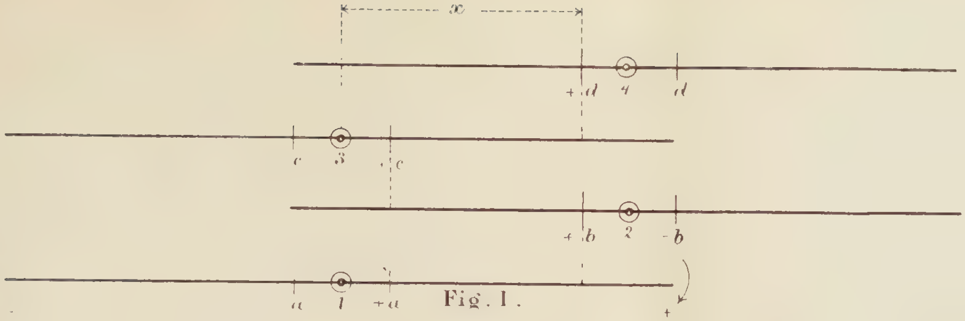

Fig. 1.

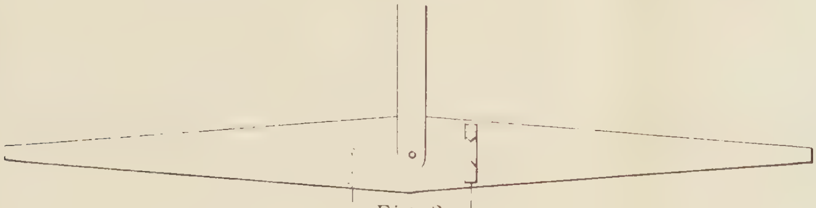

Fig. 2

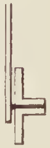

Fig. 5.

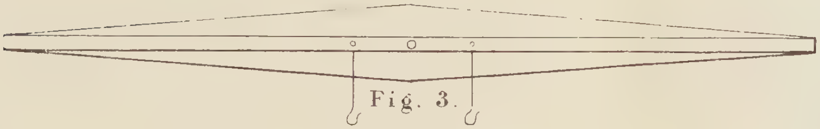

Fig. 3.

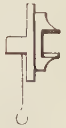

Fig. 6.

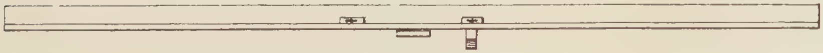

Fig. 4.

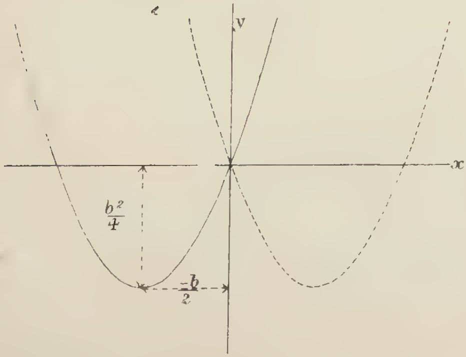

Fig. 7.

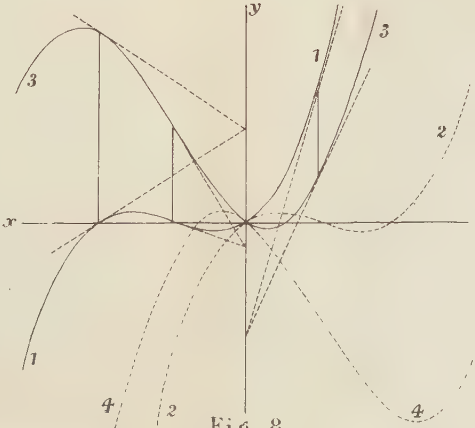

Fig. 8.

Mintern Bros. lith.

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1885–86.

February 14th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following was elected a Member of the Society:—

Prof. George Fuller.

The following communications were made:—

"On a Line and Area Divider." By Miss Sarah Marks.

"On Recent Additions to his Integrating Anemometer." By Mr. Walter Baily.

February 28th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following were elected Members of the Society:—

Mr. Osbert Chadwick, late R.E.; Mr. G. R. Begley.

The following communications were made :—

"Notes on the Use of Nicol's Prisms." By Mr. J. C. McConnel.

"On Polarizing Prisms." By Mr. H. G. Madan.

"On the most Economic Potential-difference to employ with Incandescent Lamps." By Profs. Ayrton and Perry.

"On the Second Law of Thermodynamics." By Mr. J. Macfarlane Gray.

March 14th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following communications were made :—

"On Recent Researches in Radiation." By Capt. Abney.

"On the Characteristic Curves of Incandescence Lamps." By Dr. J. A. Fleming.

"On Lecture-Experiments in Spectrum Analysis." By Mr. E. Cleminshaw.

"On Sir Wm. Thomson's Quadrant-Electrometer." By Dr. J. Hopkinson.

March 28th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following was elected a Member of the Society :—

Mr. F. B. Hawes.

The following communications were made :—

"On Calculating Machines, illustrated by a Collection of Ancient and Modern Machines, lent by the owners for the occasion." By Mr. Joseph Edmondson.

"On the Structure of Mechanical Models illustrating some of the Properties of the Æther." By Prof. G. F. Fitzgerald.

Instruments and Apparatus were exhibited by Col. E. D. Malcolm, Mr. W. F. Stanley, Mr. Conrad C. Cooke, Mr. Hilger, and Dr. W. M. Watts.

April 25th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following communications were made :—

"On the Theory of Illumination in a Fog." By Lord Rayleigh.

"On a Monochromatic Telescope, with Applications to Photometry." By Lord RAYLEIGH.

"On the Self-Regulation of the Compound Dynamo." By Prof. A. W. RÜCKER.

"On the Determination of the Heat Capacity of Thermometers." By Mr. J. W. CLARK.

"On Mr. Sugg's Inclined Photometer." By Mr. LIVINGSTONE.

May 9th, 1885.

The Meeting was held in the Physical Lecture Room, Clifton College, Bristol.

Dr. F. GUTHRIE, President, in the Chair.

The following were elected Members of the Society:—

Mr. E. F. HERROUN; Mr. ARTHUR L. SELBY, B.A.;
Mr. E. CLEMINSHAW, M.A.

The following communications were made:—

"On Evaporation and Dissociation." By Prof. W. RAMSAY and Dr. S. YOUNG.

"On a Model illustrating Propagation of the Electro-magnetic Wave." By Prof. S. P. THOMPSON.

"On a Self-recording Stress and Strain Indicator." By Prof. H. S. HELE SHAW.

"On the so-called Silent Discharge in Ozone Generators." By Mr. W. A. SHENSTONE.

May 23rd, 1885.

Dr. F. GUTHRIE, President, in the Chair.

The following was elected a Member of the Society:—

Mr. A. H. FISON, D.Sc.

The following communications were made:—

"Experiments showing the Variations caused by Magnetization in the length of Iron, Steel, and Nickel Rods." By Mr. SHELFORD BIDWELL.

"On the Spectral Image produced by a Rotating Vacuum-Tube." By Mr. SHELFORD BIDWELL.

"On the Action of Light in Diminishing the Electrical Resistance of Selenium, and on a Method of producing similar effects with Sulphur." By Mr. SHELFORD BIDWELL.

"On certain cases of Electrolytic Decomposition." By Mr. J. W. CLARK.

"Notes on Electrical Symbols." By Mr. J. MONRO.

June 13th, 1885.

Dr. F. GUTHRIE, President, in the Chair.

The following communications were made:—

"On the Comparison of Mercury Standards of Resistance issued by M. Mascart with those of the British Association." By Mr. R. T. GLAZEBROOK.

"On the Winding of Voltmeters." By Profs. AYRTON and PERRY.

"On the Manner in which Light affects the Resistance of Selenium and Sulphur Cells." By Mr. SHELFORD BIDWELL.

"On the Error involved in Prof. Quincke's Method of Calculating Surface-Tensions from the Dimensions of Flat Drops and Bubbles." By Mr. A. M. WORTHINGTON.

June 27th, 1885.

Dr. F. GUTHRIE, President, in the Chair.

The following were elected Members of the Society:—

THOMAS HANDS, M.A.; FRANCIS HENRY NALDER; Prof. W. RAMSAY, Ph.D.; F. W. SANDERSON, B.A.; W. A. SHENSTONE.

The following communications were made:—

"On the Specific Refraction and Dispersion of the Alums." By Dr. J. H. GLADSTONE.

"On a Form of Standard Daniell Cell and its Application for Measuring Large Currents." By Dr. J. A. FLEMING.

"On the Phenomenon of Molecular Radiation in Incandescent Lamps." By Dr. J. A. FLEMING.

"On Problems in Networks of Conductors." By Dr. J. A. Fleming.

"On Lecture-Experiments in Colour Mixtures." By Capt. Abney.

"On the Stream-Lines of Moving Vortex Rings." By Prof. O. J. Lodge.

"On the Thermoelectric Position of Carbon." By Mr. J. Buchanan.

"Further Experiments on Sulphur Cells." By Mr. Shelford Bidwell.

November 14th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following communications were made:—

"On Testing Thermometers at the Melting-point of Mercury." By Mr. G. M. Whipple.

"On the Electromotive Force of certain Tin Cells." By Mr. E. T. Herroun.

"On the Law of the Electromagnet, and the Law of the Dynamo." By Prof. S. P. Thompson.

November 28th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following was elected a Member of the Society:—

Thomas Holmes Blakesley, M.A., M.I.C.E.

The following communications were made:—

"On the Calibration of Galvanometers by a Constant Current." By Mr. T. Mather.

"On a new Driving Clock-work of Isochronous Motion." By Mr. A. Hilger.

"On a new Direct-Vision Spectroscope." By Mr. A. Hilger.

"On a Machine for the Solution of Equations." By Mr. C. Vernon Boys.

"On a Machine for the Solution of Cubic Equations." By Mr. H. H. Cunynghame.

"On some Thermodynamic Relations." By Drs. Ramsay and Young.

December 12th, 1885.

Dr. F. Guthrie, President, in the Chair.

The following were elected Members of the Society:—

Charles Frederic Casella; Prof. T. E. Thorpe, F.R.S.

The following communications were made:—

"On a Magneto-electric Phenomenon." By Mr. Wyatt.

"On some Thermodynamic Relations." By Drs. Ramsay and Young.

January 23rd, 1886.

Dr. F. Guthrie, President, in the Chair.

The following communications were made:—

"Note on a Paper by Prof. W. Ramsay and Dr. S. Young 'On some Thermodynamic Relations.'" By Profs. Ayrton and Perry.

"Note on a Paper by Mr. J. W. Clark on the Determination of the Heat-Capacity of a Thermometer." By Mr. A. W. Clayden.

"Note on some Organic Substances of High Refracting Power." By Mr. H. G. Madan.

"On an Apparatus consisting of a Vibrating Metal Strip with associated Air-Column, illustrating the Motion of Matter at the Nodes." By the President.

Annual General Meeting.

February 13th, 1886.

Dr. F. Guthrie, President, in the Chair.

The following Report of the Council was read by the President:—

In presenting their Annual Report the Council are able to congratulate the Society on the steady position which the work of the Society has attained. On looking through the 'Proceedings' it will be seen that the first five volumes include the work of nine years; but that the sixth volume is entirely occupied with the work of one year, viz. the Session which commenced with the Ordinary Meeting held on February 9, 1884, and terminated with the Annual General Meeting held on February 14, 1885. The Council, feeling that it was now possible to fix definitely the times at which the Parts of the 'Proceedings' should be issued to Members, have decided that these parts should be issued quarterly, viz. in July, October, January, and April.

It is intended that these four Parts should form one volume, so that in future each volume of 'Proceedings' should contain the work of one Session. The first three Parts of Vol. VII. have already been issued in accordance with this arrangement. Among the various papers of interest which have been presented to the Society, the Council desire to call special attention to the valuable paper by Prof. Rücker on the Compound Dynamo.

The Society held a Meeting last summer in Bristol, at which the Educational Institutions of the town were visited; and the Council desire to express their thanks to the Members and other friends residing at Bristol for the efficiency of the arrangements which were made.

In the Annual Report of last year it was mentioned that space had been allotted to the Physical Society in the International Exhibition of Inventions. Cases were lent by the Science and Art Department, and other cases were well filled by Apparatus lent by Members of the Society. Besides the Apparatus exhibited in the space allotted to the Physical Society, many things were shown by our Members in other parts of the Exhibition. The Council desire to record their thanks to the Council of the Inventions Exhibition for the allotment of space and for other facilities.

The hope expressed in last year's Report that the second volume of the reprint of Dr. Joule's papers would be issued in the current year has not been realized. More than half the volume, however, is in type, and Dr. Joule expects to be able to supply the rest of the copy at an early date; the Council therefore feel confident that they will be able to complete the book this year.

The Library now consists of 512 separate works; 220 Transactions of Societies, Proceedings, &c.; 130 separate Memoirs.

Many valuable gifts have been received from the various learned Societies and Authors at home and abroad. Amongst which may be mentioned:—

Journal of the Society of Arts.
Reports of the Natural History Society, Canada.
Catalogue of Canadian Plants. Part II. Gamopetalæ.
Clark's Transit Tables.
Proceedings of the Cambridge Philosophical Society.

Journal and Proceedings of the Royal Society of New South Wales.

Proceedings of the Royal Society of London.

Institution of Mechanical Engineers, Proceedings.

Proceedings of the Royal Society of Edinburgh.

Abhandlungen des Universität zu Tôkiô.

Transactions of the Seismological Society of Japan.

Recherches sur les Lois des l'écoulement et du choc des Gaz. G.-A. Hirn.

Chemical and Physical Journal, St. Petersburg.

Quarterly Journal of the Royal Meteorological Society.

Norwegian North-Sea Expedition. Zoologi. Crustacea: 1^{A}, 1^{B}.

Papers on Technical Education, Applied Science Buildings, Fittings, and Sanitation. E. C. Robins.

Electrical Review.

Observations and Researches made at the Hong Kong Observatory in 1884, by Doberck.

Magnetical and Meteorological observations, Government Observatory, Bombay.

Circulars of information, Bureau of Education, Washington.

Radcliffe Library, Oxford. Catalogue of Additions.

Mauritius Meteorological Observatory Reports.

The Council have considered that the Library might be made more useful to Members by the institution of a Librarian, who should be applied to in all matters relating to the Library, and they are glad to announce that Mr. Boys has kindly consented to perform the duties of this office.

The Council regret to announce the deaths of three Members—Prof. T. Andrews, Prof. J. W. Clark, and Mr. W. Ladd.

Thomas Andrews was born at Belfast on the 19th December, 1813. After receiving his early education in his native town, he proceeded to Glasgow, where he studied Chemistry under Dr. Thomas Thomson; and thence to Trinity College, Dublin, where he distinguished himself both in Classics and Science. He continued his Chemical Studies in Paris under Dumas. In 1835 he obtained his Medical Degree in the University of Edinburgh; and returning to Belfast, in which he thenceforth spent his life, he gained a highly successful practice as a Medical man, and devoted his leisure time to scientific research. Dr. Andrews gave up his medical practice, and also the chemical lectureship in the Royal Belfast Academy, upon his appointment in 1845 to the vice-presidentship of Queen's College (or, as it was then called, "the Northern College"), Belfast.

The duty of advising the Government in maturing a plan for the Queen's Colleges and of getting them into working order fell upon the Presidents and Vice-Presidents, and in the performance of this duty Dr. Andrews took a prominent part. He was himself appointed Professor of Chemistry at Belfast, and held the office for 30 years. In 1849 he was elected a Fellow of the Royal Society. In 1852 he was President of the Chemical Section of the British Association at Belfast, and occupied the same position in Edinburgh in 1871. Five years later he was President of the Association at Glasgow. In 1879 he resigned his appointments at Queen's College, and led a very retired life until his death on November 26th, 1885. Dr. Andrews was remarkable for his originality as a scientific thinker, and for the extreme delicacy and care of his experimental work. His early work earned from the Royal Society one of the Royal Medals as far back as 1844; but the importance of his discoveries continually increased. One of his great investigations was the determination of the heat evolved during chemical action. He here led the way in a study, the great value of which is now thoroughly recognized, and his earlier papers on this subject were soon followed by those of Favre and Silberman. Another important paper was that communicated to the Royal Society on the subject of ozone, in which he showed by a series of remarkably delicate experiments that ozone was oxygen in an altered or allotropic condition. The study of ozone was continued by Andrews and Tait, and their paper on the subject published in the Phil. Trans. 1860 announced the discovery of its constitution.

The crowning work of Dr. Andrews's life was his research into the continuity of the Liquid and Gaseous states of Matter, which employed his highest powers of experiment, and which must remain to all time as one of the most marked steps in the progress of Physical Science.

William Ladd, who was one of the original Members of this Society, was born at Deal in 1815 and came to London at the age of 14. He took a prominent part in bringing Electricity into practical use, and was well known as the constructor of the largest induction-coils and electrical apparatus. In 1867 Mr. Ladd brought forward a dynamo machine, in which many of the difficulties in constructing dynamos for practical use were overcome. In 1878 he introduced from America the Wallace Farmer system of electric lighting, and effected in it considerable improvements. One of the earliest installations of electric lighting, viz. that at the Liverpool

Street Terminus of the Great Eastern Railway, was made under Mr. Ladd's supervision.

Mr. Ladd took an active part as director in the promotion and management of the Anglo-American Brush Electric Light Corporation (Limited), and also of the Electric Power Storage Company.

He died on April 16, 1885.

The Council are indebted to Dr. O. J. Lodge for the following memoir.

J. W. CLARK, the late Assistant Professor of Physics in University College, Liverpool, was the son of Mr. Joseph Turner Clark, of Southampton, member of the Society of Friends, and was born on 1st July, 1856. From his earliest childhood he is reported to have been studious, and to have taken many prizes in scientific subjects at Weston School, whither he went for two years, at the age of 15. His father destined him for business, but he managed to attend evening classes at the Hartley Institution, Southampton; and on the occasion of a lecture by Professor Barrett in that town he made earnest appeal to the professor to use his influence and help him towards a scientific career. He was permitted to accompany Mr. Barrett on a lecturing tour in the North, and afterwards to enter as a student in the Royal College of Science, Dublin, 1874 and 1875. Professor Barrett may be thus considered to have commenced and directed Clark's scientific development, and speaks of him as follows:—"Clark was the most enthusiastic student of science I have ever known. It was very difficult to get him away from the laboratory, or from his books, at night. His only fault was a tendency to discursive reading and a dislike to study those things for which he had no taste. He made a series of investigations when with me on the effect of high pressure and temperature on certain of the chemical and physical properties of liquids, and nearly blew his eye out by an explosion: he was at work again before his eye was healed."

Under Professor Barrett he acquired considerable manipulative dexterity and experimental facility, which were extended and widened by his next two years of study at Heidelberg. For both Professor Bunsen and Professor Quincke he retained the warmest attachment, and indeed veneration. The life in Germany deeply modified his whole tone of thought. He made many friends there, and became to all intents and purposes a German student.

The experience gained under Professors Quincke and Bunsen remained probably the dominant influence of Clark's life; and to the end, *i. e.* some nine years after, he was still engaged in researches the germs of which had been planted at Heidelberg.

From Germany he proceeded, as Demonstrator in Chemistry and Physics, to the Royal Indian Engineering College at Coopers Hill, under Professor McLeod. While at Coopers Hill he continued several researches, and published some of them; but publication was his weak point, at least in the opinion of the writer. By the time the research was sufficiently complete for him to think of publication, the vivid interest of it seemed to have evaporated, or perhaps something else began to fill his mind, and accordingly what he wrote was somewhat perfunctorily put together.

The writer is convinced, however, that statements made in his papers may be relied on for most careful accuracy; and nothing vexed him more than to hear or see a statement of his loosely questioned or cavilled at. Yet the casual or parenthetical aspect of some of his statements, suggesting to a reader that they were perhaps mere hasty surmises, often led to criticisms of this sort; whereas the statement really embodied, and was intended to express and convey the gist of, the labour of weeks.

After some four years of work at Coopers Hill he resigned his post and lived at home for eighteen months; when, as the result of some scientific correspondence between him and the writer, he came to University College, Liverpool, in the Spring of 1883—first, rather informally, afterwards as official Demonstrator in Physics, and finally as Assistant Professor. Into the organization of the incipient Physical Laboratory, then going on, he threw himself heart and soul; working at all out-of-the-way hours, and far into, or even through, the night. These violent exertions of course told upon his health, and necessitated periods of quiescence. Often remonstrated with, he replied that he could not help it, that he must work furiously when the fit seized him; and so, indeed, it seemed.

Such absorbing devotion to experimental research the writer never before encountered; and no less remarkable were the ready ingenuity and intuitive skill with which difficulties were encountered and foreseen. A difficulty, or a lack of an experimental kind, instantly seemed to brighten up every faculty of the man's nature; and, sometimes at once, sometimes after a night's brooding, sketches of all manner of contrivances were thrown off, and the selected one soon put into execution.

Had he been a less reckless worker, and thus kept in better health, and had there been a place in England where a man's bodily wants were provided for, leaving him free to pursue research without let or hindrance, he would have done great things; but there is no such place; though, indeed, the writer tried his very best to create such a place for him at Liverpool, and in this attempt was nobly seconded by the Principal and Council of the College: and perhaps if there had been, the health difficulty would have interfered. However that may be, he showed signs of breaking down at the end of the long vacation—which, it is needless to say, is only named "holiday" in the sense of being more deeply and continuously consecrated to the diviner portion of one's work—and he was granted one term's leave of absence for foreign travel. He returned to his old and well-loved haunts at Heidelberg, saw Prof. Quincke and the laboratory once more, and died there at the early age of 29, on the 27th of last October.

A sad end to a most promising career! His loss to the Physical department of the College is irremediable; the loss to science, though unfelt, is probably not less essentially severe. To the writer, the loss of his invigorating criticism, unique scientific culture, and strong personal friendship is a painful blow.

The Treasurer made the Financial Statement given below.

The Society then proceeded to the election of Officers and other Members of Council for the ensuing year. The Council was constituted as follows:—

President.—Prof. BALFOUR STEWART, LL.D., F.R.S.

Permanent Vice-Presidents.—Dr. J. H. GLADSTONE, F.R.S.; Prof. G. C. FOSTER, F.R.S.; Prof. W. G. ADAMS, M.A., F.R.S.; Sir WILLIAM THOMSON, D.C.L., LL.D., F.R.S.; Prof. R. B. CLIFTON, F.R.S.; Dr. F. GUTHRIE, F.R.S.

Other Vice-Presidents.—Prof. W. E. AYRTON, F.R.S.; SHELFORD BIDWELL, M.A., LL.B.; Prof. H. MCLEOD, F.R.S.; Prof. W. CHANDLER ROBERTS-AUSTEN, F.R.S.

Secretaries.—Prof. A. W. REINOLD, M.A., F.R.S.; WALTER BAILY, M.A.

Treasurer.—Dr. E. ATKINSON.

Demonstrator.—C. VERNON BOYS.

Other Members of Council.—CONRAD W. COOKE; Prof. G. FORBES, F.R.S.E.; Prof. F. FULLER, M.A.; R. T. GLAZEBROOK, M.A., F.R.S.; J. HOPKINSON, D.Sc., F.R.S.; Prof. J. PERRY, F.R.S.; Prof. J. H. POYNTING, M.A.; A. W. RÜCKER, M.A., F.R.S.; Prof. S. P. THOMPSON, D.Sc.; C. ALDER WRIGHT, D.Sc., F.R.S.

Votes of thanks were passed to the Lords of the Committee of Council on Education; to the PRESIDENT and other OFFICERS of the Society; and to the AUDITORS.

The Treasurer in Account with the Physical Society, from January 1st, 1885, to December 31st, 1885.

Dr.	£	s.	d.	£	s.	d.
Balance in Bank, January 1, 1885				73	11	7
" due by Treasurer, Jan. 1, 1885				1	1	7
14 Entrance-Fees	14	0	0			
1 Subscription for 1881	1	0	0			
2 Subscriptions for 1882	2	0	0			
3 " 1883	3	0	0			
14 " 1884	14	0	0			
119 " 1885	119	0	0			
4 " 1886	4	0	0			
Life Composition	50	0	0			
				207	0	0
One year's Dividend on £400 Furness 4 per cent. Debenture Stock, less Income Tax	15	11	0			
One year's Dividend on £460 Midland 5 per cent. Preference Stock, less Income Tax	22	7	1			
One year's Dividend on £200 Metropolitan Board of Works Stock, less Income Tax	6	15	8			
One year's Dividend on £200 Lancaster Corporation Stock	7	15	4			
				52	9	1
Sale Account:—						
Joule (vol. i.)	25	18	0			
Wheatstone	3	10	0			
Proceedings	3	5	0			
	32	13	0			
Commission	3	5	3			
				29	7	9
Balance due to Treasurer				5	10	4
				£369	0	4

Cr.	£	s.	d.	£	s.	d.
Taylor and Francis:—						
Proceedings, vol. vi. (parts 3 & 4), vol. vii. (parts 1 & 2)	130	18	6			
Postage and addressing	14	18	9			
Members' separate copies	25	19	0			
Miscellaneous printing	22	17	2			
				194	13	5
Reports of Meetings				17	16	0
Williams and Norgate				3	4	0
Bookbinding				17	7	7
Translation of Volta				2	10	0
Attendance at Meetings and incidental expenses				7	10	1
Petty Cash:—						
Mr. Reinold	1	11	8			
Mr. Baily	0	18	0			
Mr. Atkinson	1	16	0			
				4	5	8
Stationery				4	13	9
Loss on Cheques				0	1	4
Balance in Bank				116	18	6
				£369	0	4

Audited and found correct,

E. ROBERT FESTING, } *Auditors.*
GEORGE FULLER,

London, February 1st, 1886.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY.

ASSETS.	£	s.	d.
Balance in Bank	116	18	6
Estimated Subscriptions due	36	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0
£460 Midland 5 per cent. Preference Stock	648	0	0
£200 Metropolitan Board of Works Stock	217	0	0
£200 Lancaster Corporation Stock	214	0	0
	£1687	18	6

LIABILITIES.	£	s.	d.
Due to Treasurer	5	10	4
Subscriptions for 1886	4	0	0
Balance	1678	8	2
	£1687	18	6

We have examined the above Account, and also the Securities at the Bank, and find the same to be correct.

E. ROBERT FESTING, } *Auditors.*
GEORGE FULLER, }

London, February 1st, 1886.

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